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EVIDENCE OF CONTAINMENT OF A 3-kev DEUTERIUM PLASMA*

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The production and the stable confinement of hot plasma remain the foremost problems of high-temperature plasma research. As was recently pointed out,[1] the observed long-time containment in a magnetic mirror field of plasmas in which the electron temperatures are in the range of 10 to 25 kev is contrary to simple theoretical predictions and is unique in laboratory experience. There is considerable speculation whether or not the more interesting plasmas in which the mean ion energy equals or exceeds the mean electron energy can also be stably confined in the same geometry. Plasmas which are produced by rapid magnetic compression[2-4] of slightly ionized gas of tens of microns pressure may have temperatures of 1 to 2 kev but they are dissipated in a few microseconds and yield little information concerning containment or stability.

It is the purpose of this Letter to report the production and containment of a plasma in which the mean ion energy is about 3 kev and the electron temperature is of the order of 50 ev. This plasma is produced by magnetic compression of a highly ionized but "colder" plasma which is injected into an initially evacuated chamber. The experiment is similar to the first two stages of a multistage magnetic compression experiment[5] reported previously. The plasma generators, the techniques of compression and transfer, and the ion energy analyzer are described in the earlier paper. In the present experiment, plasma is injected into a uniform base field of 600 gauss at a point 130 in. from a dc barrier field. The leading front of the plasma burst is trapped between the dc barrier and a pulsed magnetic field where it is compressed and heated by the rising magnetic field. It is possible to adjust the relative values of the barrier and pulsed fields so that the entire plasma is transferred through the barrier; or, alternatively, between 30 and 50% of the compressed plasma can be retained in a mirror containment region where the mirror ratio is 1.08. It is possible to determine the ion species, the ion energy distribution, and the total positive charge in the compressed and transferred plasma.

The conditions in the injected plasma are known to be: density $= 5\times10^{12}$ ions/cc, mean ion energy = 850 ev, mean ion rotational energy about field lines = 50 to 60 ev, and electron temperatures = 3 to 7 ev. For these conditions the 50-μsec compression time is 23 times the ion cyclotron period and about 0.1 of the ion relaxation time.[6] Consequently, the ions are expected to behave adiabatically. That is, the final rotational energy should be equal to the product of the initial rotational energy and the magnetic compression ratio α. (α is essentially the ratio of the dc barrier field to the magnitude of the field at the injection point.) Initially the relaxation time for the electrons is only 1 μsec, so that they behave as a gas with 3 degrees of freedom and should gain energy from the pulsed field as $\alpha^{2/3}$. Thus, the final electron temperature would be 20 ev if they gain energy only from the field. However, due to energy transport from the energetic ions the electron temperature will prob-

ably be of the order of 50 ev.

Adiabatic heating of the ions has been confirmed within the accuracy of these experiments. For example, in a compression of $\alpha = 33$ the mean ion energy was found to be ~3000 ev compared to the predicted 2700 ev. Comparison of the D^+ energy distribution before and after this compression, Fig. 1 curves I and II, respectively, clearly shows the heating. An estimate of the ion energy distribution at the end of the compression (70 μsec) in the plasma which is trapped between the magnetic mirrors is shown in Fig. 1 curve III. This distribution is the difference between distributions for a totally transferred plasma and the transferred fraction of the plasma in the containment experiment. In the latter case the mean ion energy in the transferred fraction was 2.8 kev, that in the contained fraction was also 2.8 kev, which agrees within experimental accuracy with the 3 kev obtained for the totally transferred plasma. At 70 μsec the conditions in the trapped plasma are estimated to be within the following ranges: mean ion energy 2.4 to 3.2 kev, total volume 1.6 to 2.4 liters, average density 10^{13} to 3×10^{13} ions/cc, electron temperature 20 to 100 ev, and plasma energy density 0.25 to 0.75% that of the magnetic field. These conditions are consistent with the observed production of 10^4 neutrons per operation. Neutron identification was made by the characteristic light pulses produced in a LiI crystal scintillator from the reaction $Li^6(n, H^3)He^4$. No evidence of large x-ray bursts which could have confused the identification was detected by a 2-in. by 2-in. NaI crystal scintillator. The absence of x rays is consistent with the low compression ratio of 30 (compression ratios of the order of 1000 are used in experiments in which energetic electrons and x rays are detected).

The observed adiabatic behavior of the ions during the slow compression is consistent with stable containment for this period, and containment for at least 900 μsec is indicated by the histogram of neutron production, Fig. 2. The initial rapid decrease in the neutron production rate is associated with the decrease in plasma density as part of the plasma is forced through the dc barrier. After 70 μsec, the decay of the production rate is probably due to the decrease in mean ion energy and density caused by the decay of the pulsed field (700 μsec characteristic time). The time history was obtained by use of a plastic scintillator to detect fast neutrons. Identification of these counts as due to neutrons was made by measurement of their absorption length in lead. The question arises for this plasma (as for all other plasmas of a few kev ion energy) whether or not the neutron production is from the plasma or due to a few high-energy ions bombarding a target of cold gas. The latter assumption is inconsistent with the 900-μsec containment time as the neutral density necessary to account for the observed rate

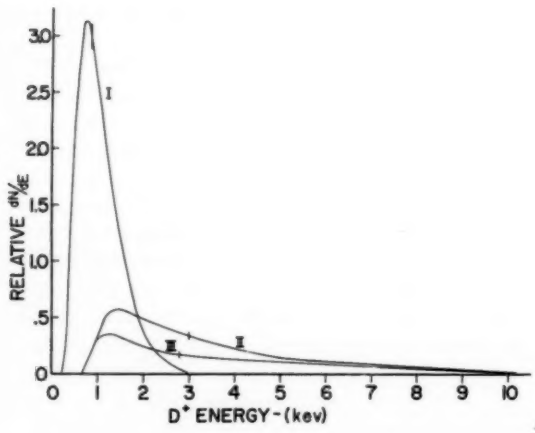

FIG. 1. D^+ energy distributions. Curve I, energy distribution of the ions in the injected plasma. Curve II, energy distribution of the ions after a compression of $\alpha = 33$. Curve III, energy distribution of the ions (at 70 μsec) in the trapped plasma. The mean ion energy for each distribution is indicated by a short vertical line.

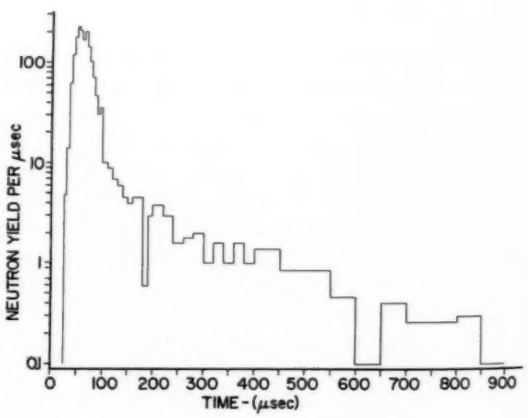

FIG. 2. Histogram of neutron production.

would, through charge exchange, lead to a much shorter lifetime of the high-energy group. It is therefore necessary to postulate containment of the observed plasma either as a target or as a reacting plasma. The evidence favors the latter interpretation. We conclude that the plasma is contained for times which exceed the e-folding time for hydromagnetic instabilities by three orders of magnitude. Furthermore, the close agreement of the observed ion energy gain to that predicted from adiabatic theory indicates that the energy transport from ions to electrons does not greatly exceed the rate calculated on the basis of Coulomb collisions.

It is a pleasure to acknowledge the many profitable discussions with R. F. Post, and to express our indebtedness to D. R. Branum, who has had the responsibility for the design, construction, and operation of the extensive electrical systems.

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DIRECT MEASUREMENT OF THE SUPERCONDUCTING ENERGY GAP

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Giaever[1] has reported experiments on the tunneling of electrons through a thin aluminum oxide layer between a film of aluminum in the normal state and a film of lead maintained either in the normal or in the superconducting state. The results were interpreted qualitatively in terms of the change in the density-of-states function at the Fermi level of lead upon passage from the normal to the superconducting state.

In this Letter, the tunneling current vs voltage characteristic of an Al-Al_2O_3-Pb sandwich is presented when both Pb and Al are superconducting. This characteristic exhibits a negative resistance region (see Fig. 1). Analysis of the problem of electron tunneling through an insulating layer between two superconductors shows that the negative resistance region is a direct consequence of the existence of an energy gap in the superconducting density-of-states function. The voltage difference between the points of maximum and minimum current which define the negative resistance region is a direct and unambiguous measure of the full energy gap of the metal with the lower transition temperature. The I - V characteristic is symmetric through the origin. The magnitude of the difference in voltage between the mid-points on the negative resistance regions for positive and for negative voltage is a direct and unambiguous measure of the full energy gap of the metal with the higher transition temperature.

Samples were prepared by evaporating an Al strip onto a glass substrate, oxidizing in air, and evaporating a Pb cross strip. The area of crossing was approximately one square millimeter; the thickness of Al_2O_3 was estimated to be about 20 A. A specially built curve tracer was used to display the I - V characteristic on an oscilloscope. The resistance of each metal was

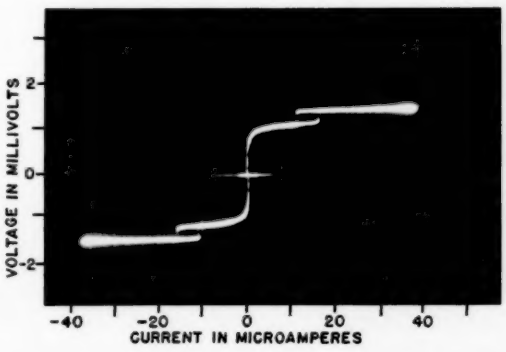

FIG. 1. Voltage vs tunneling current for an Al-Al_2O_3-Pb sandwich at 1°K.

also measured. The sample was in contact with liquid helium at 4.2°K and below. Temperature was determined from the liquid helium vapor pressure or from a carbon resistance thermometer.

The tunneling current vs voltage characteristic was measured at several temperatures for a number of Al-Al_2O_3-Pb sandwiches. The experimental curves for both metals normal or for Pb alone superconducting essentially duplicated those of Giaever. As shown in Fig. 1, new results were obtained when both metals were superconducting. In particular, over a certain region, the current decreased as the voltage increased, i.e., the I-V characteristic exhibited a negative resistance region. This region first appeared at the Al transition temperature and increased in voltage amplitude as the temperature was reduced. When the Al was brought into the normal state by an external magnetic field, the negative resistance disappeared.

To account for these results, consider the expression for the tunneling current. For small voltages the tunneling probability can be taken as constant. Then the one-way tunneling current is proportional to an integral over all energies of the product of the number of electrons in one metal by the number of unoccupied states (holes) in the other metal at the corresponding energy. The net current is given by the difference in the opposed one-way currents. With the energy zero at the Fermi level for metal 1, and with metal 1 at a positive potential,

$$I\propto\int\{\rho_2(E-V)f(E-V)\rho_1(E)[1-f(E)] - \rho_2(E-V)[1-f(E-V)]\rho_1(E)f(E)\}dE. \quad (1)$$

With all energies measured in units of kT, V is the energy equivalent of the applied voltage, ρ_1 is the density-of-states function for metal 1, ρ_2 is that for metal 2, and $f(E)$ is the Fermi function,

$$f(E)=1/(1+e^{E}). \quad (2)$$

Equation (1) reduces to

$$I=\text{const}\times\int\rho_2(E-V)\rho_1(E)\{f(E-V)-f(E)\}dE. \quad (3)$$

To proceed further, the integrals must be evaluated numerically. Clearly the shape of the I-V characteristic is intimately connected with the density-of-states function and may be used to obtain that function. Detailed analysis, including machine computation, is reserved for future publication. However, initial results show good agreement, provided the Fermi function is taken into account. In Fig. 2, experimental I-V curves for an Al-Al_2O_3-Pb sandwich at 4.2°K and 2.1°K, hence for Pb alone superconducting, are compared with points calculated using Eq. (3). A constant value was assumed for the density-of-states function of aluminum, and for that of lead the Bardeen, Cooper, and Schrieffer (BCS) expression[2] was used; namely,

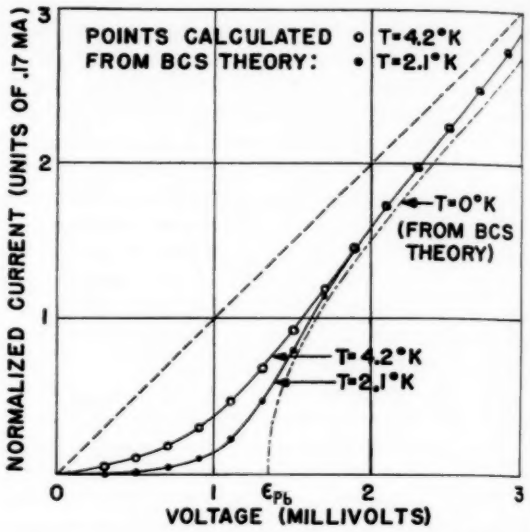

FIG. 2. Experimental curves at 4.2°K and 2.1°K for an Al-Al_2O_3-Pb sandwich compared with points calculated using the BCS density-of-states function for superconducting lead. The value used for the energy gap of lead, $4.35kT_c$, is that measured from experimental curves taken at lower temperatures when both Al and Pb are superconducting. Also shown is the limiting expression at $T=0$°K calculated for one metal normal, $I\propto(V^2-\epsilon_{Pb}^2)^{1/2}$.

$$\rho(E)=N(0)\left|\frac{E}{[E^2-\epsilon(T)^2]^{1/2}}\right|, \quad |E|\geq\epsilon(T)$$
$$\rho(E)=0, \quad |E|<\epsilon(T) \quad (4)$$

where $N(0)$ is the density-of-states at the Fermi level in the normal state and $2\epsilon(T)$ is the temperature-dependent energy gap. The value used for the energy gap of Pb was that measured by the technique to be described below. At 2.1°K the full limiting value was used; at 4.2°K a correction factor of 0.93 was applied as obtained from BCS. The calculated points are sensitive to the value of the gap; use of the value $3.5kT_c$

destroys the good agreement obtained with the measured value. The data are normalized so that the characteristic approaches a 45° slope at higher voltages. Also included in the figure is the limiting form of the characteristic at $T = 0°K$, $I \propto (V^2 - \epsilon_{Pb}^2)^{1/2}$. This curve would be approached were a metal that remained normal used in place of Al in forming the sample.

It remains to account for the negative resistance region and to demonstrate that it is a direct measure of the superconducting energy gap. Fig. 3 sketches the density-of-states function and the filled states in a small energy interval about the Fermi level for a sandwich formed of two superconducting metals separated by a thin insulating layer. The energy gap and peaking of states on both edges of the gap are indicated. The sketch is not to scale but represents a sandwich formed from a high transition temperature metal and a low transition temperature metal. In the absence of an applied voltage, the two Fermi levels are at the same energy. Applying a voltage is equivalent to sliding one density-of-states curve with respect to the other. Clearly the number of unoccupied states in metal 1 into which electrons of metal 2 may tunnel increases with voltage until the left-hand edges of each gap coincide. Similarly, the number of electrons in metal 1 decreases with voltage until the left-hand edges of each gap are within ϵ_1 or so of coinciding. Over this last interval the number of electrons in metal 1 increases because of the peaking of states; however, the product of electrons in metal 1 by holes in metal 2 is still smaller than the product of electrons in metal 2 by holes in metal 1 because of the Fermi function acting on two different gaps. The result is a net positive current from metal 1 to metal 2 which reaches a maximum at a voltage

$$V_{max} = \epsilon_2 - \epsilon_1. \tag{5}$$

As the voltage is increased further the left-hand edge of the electron distribution for metal 2 passes into a region where there are no states available. Hence there is a decrease in the current as the voltage increases; this is the negative resistance region. Only when the voltage exceeds a value

$$V_{min} = \epsilon_2 + \epsilon_1, \tag{6}$$

at which the left-hand edge of the gap for metal 2 coincides with the right-hand edge of the gap for metal 1, does the current increase again. [Note that at $T = 0°K$ there would be no negative resistance region; the current would remain zero up to a voltage given by Eq. (6).] From Eqs. (5) and (6) it is apparent that the voltage difference between the points of maximum and minimum current that define the negative resistance region is a direct measure of the full energy gap, $2\epsilon_1$, for metal 1. Knowing ϵ_1, ϵ_2 is immediately determined. Uncertainty in the location of the origin may be eliminated by using the symmetry of the characteristic and measuring the full gap, $2\epsilon_2$, for metal 2 as the difference in voltage between the mid-points of the negative resistance regions for positive and negative voltages.

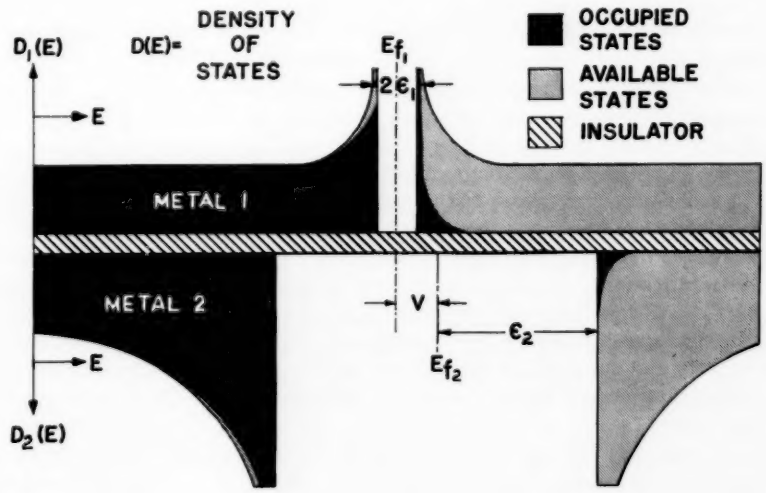

FIG. 3. The density-of-states function and the filled states sketched in a small energy interval about the Fermi level for a sandwich formed of two different superconductors separated by a thin insulating layer. A positive voltage, V, is applied to metal 1.

Using these relations, the value of the gap for Pb was determined to be $2\epsilon_{Pb} = (4.35 \pm 0.10)kT_c$, independent of temperature in the interval 0.8°K-1.2°K. The measured gap for Al was $2\epsilon_{Al}(1°K) = (1.8 \pm 0.3)kT_c$ and $2\epsilon_{Al}(0.8°K) = (2.3 \pm 0.3)kT_c$. The values 7.2°K and 1.2°K were used for the transition temperatures of Pb and Al, respectively. Using the temperature dependence of the gap given by BCS, these values imply a limiting gap for Al at absolute zero of $2\epsilon_{Al} = (2.7 \pm 0.3)kT_c$.

It is to be emphasized that these values were obtained using Al films that exhibited broad superconducting transitions in both temperature and magnetic field. The values of the energy gap, especially for Al, should therefore be regarded as provisional. Experiments are in progress to obtain the detailed temperature and field dependence of the superconducting energy gap for bulk Al and will be reported at a future date.

The authors would like to acknowledge helpful discussions with R. S. Davis and thank M. L. Cohen for the loan of equipment. The cooperation and advice of P. F. Strong with the computations are greatly appreciated.

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ELECTRON TUNNELING BETWEEN TWO SUPERCONDUCTORS

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When two metals are separated by a thin insulating film, electrons can flow between the two conductors due to the quantum mechanical tunnel effect. If a small potential difference is applied between the two metals, the current through the film will vary linearly with the applied voltage, as long as the density of states in the two metals is constant over the applied voltage range,[1] as it is for most metals. In a superconductor, however, the density of states changes rapidly in a narrow energy range centered at the Fermi level, so that the voltage-current characteristic becomes nonlinear.[2] It is relatively easy to correlate the change from linearity with the variation in the density of states. Under the assumption that the tunnel current is proportional to the density of states, the current between normal and superconducting metals is in good agreement with the density of states calculated for a superconductor by the Bardeen-Cooper-Schrieffer[3] theory.[4]

A more direct measure of the energy gap is possible when electrons tunnel between two superconductors, as may be understood from a one-particle model of a superconductor as shown in Fig. 1. All the observed phenomena of tunneling into superconductors can be understood both qualitatively and quantitatively if we are willing to accept this model, which actually guided the experiments.

The samples were prepared by vapor-depositing aluminum on ordinary glass slides and allowing the surface of the aluminum film to oxidize. After a suitable oxide layer had formed, lead, indium, or aluminum was vapor-deposited over it to form a metal-oxide-metal sandwich. The oxide layer is thought to be 15-20 A thick.

In Fig. 2 are shown some typical voltage-current characteristics for the three different metal-oxide-metal sandwiches tested. The voltage scale is in millivolts while the current scale is in arbitrary units. An X-Y recorder was used in taking the data. The sandwich involving the lead behaves exactly as predicted from the model in Fig. 1. Actually to obtain the curve it was necessary to shunt the sample with an RC network to damp out self-induced oscillations. The sandwich involving the indium shows basically the same characteristics, although for indium the unstable region was not traced out. Also, as is apparent from the low-current behavior of this sample, the oxide film is pierced by a superconductive bridge. When the current is increased the bridge goes normal, and its conductivity is too low to affect the general characteristics of the tunneling. When the current is decreased, the bridge remains normal at a lower current due to Joule heating. Finally the sandwich involving the aluminum is a little different, as here the energy gaps on either side of the oxide are equal, and at this temperature the Fermi tail is of the same order of magnitude as half the gap width.

The energy gaps obtained from these experi-

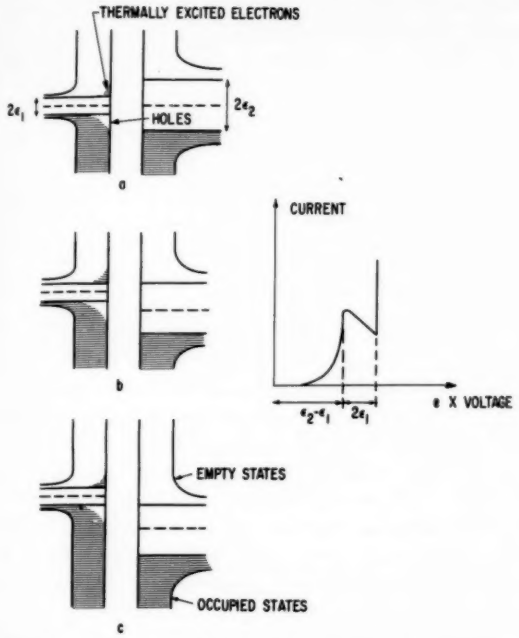

FIG. 1. Analysis of the current-voltage characteristic of two superconductors separated by a thin film. (a) The two superconductors with no voltage applied. Thermally excited electrons and holes are shown for the smaller gap, while for the larger gap there will be relatively few thermally excited electrons. (b) When a voltage is applied, a current will flow and will increase with voltage, because more and more of the thermally excited electrons in the left-hand superconductor are raised above the forbidden gap in the right-hand superconductor, and can tunnel. When the applied voltage corresponds to half the difference of the two energy gaps, $\epsilon_2 - \epsilon_1$, it has become energetically possible for all the thermally excited electrons to tunnel through the film. (c) When the voltage is increased further, only the same number of electrons can tunnel, and since they now face a less favorable (lower) density of states, the current will decrease. Finally, when a voltage greater than half the sum of the two energy gaps, $\epsilon_2 + \epsilon_1$, is applied, the current will increase rapidly because electrons below the gap can begin to flow.

ments are

$$2\epsilon_{Pb} = (2.68 \pm 0.06) \times 10^{-3} \text{ electron volt},$$

$$2\epsilon_{In} = (1.05 \pm 0.03) \times 10^{-3} \text{ electron volt},$$

$$2\epsilon_{Al} = (0.32 \pm 0.03) \times 10^{-3} \text{ electron volt},$$

For indium and lead, these gaps should not be significantly different at absolute zero, and we

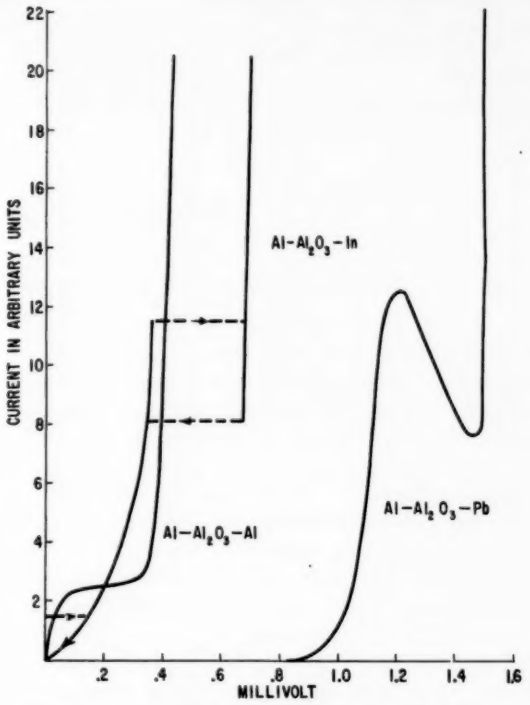

FIG. 2. Characteristic curves for tunneling between two superconductors, showing agreement with the analysis of Fig. 1. The curves Al-Al_2O_3-In and Al-Al_2O_3-Al are taken at $T \sim 1.1°K$, while the curve Al-Al_2O_3-Pb is at $T \sim 1.0°K$.

obtain

$$2\epsilon_{Pb} = (4.33 \pm 0.10)kT_c,$$

$$2\epsilon_{In} = (3.63 \pm 0.10)kT_c,$$

where T_c is the bulk transition temperature. This direct measurement of the energy gap for lead is a little smaller than what was obtained by fitting the experimental results with the BCS theory, where the best fit was obtained with an energy gap of $4.5kT_c$.[4] It should be noted that quite a large spread in the transition temperature of the aluminum films has been found; a transition temperature as high as 1.8°K has been observed. Whether this is true for the lead and indium films as well is not known.

I wish to thank J. C. Fisher and C. P. Bean for their interest and encouragement, L. B. Nesbitt for his advice concerning low-temperature physics, and K. R. Megerle for his help in performing

the experiments.

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IONIC HALL EFFECT IN SODIUM CHLORIDE*

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High-temperature ionic Hall effect measurements in large single crystals of pure sodium chloride were undertaken in order (a) to demonstrate the existence of the Hall effect due to the motion of ionic charge carriers, and (b) to provide an independent measurement of the temperature dependence of the mobility of the ionic charge carriers in sodium chloride.

The ionic Hall effect has previously never been observed in a solid, although an upper limit for the effect in NaCl has been set.[1]

Two ionic drift-mobility measurements in NaCl exist, but neither measurement was carried out near the melting point.[2,3] Both measurements used the impurity-doping method of Koch and Wagner[4] and are subject to the uncertainties implicit in this method.[5] The two experiments, which measured the drift mobility of the sodium ions, gave results which are in only rough agreement with each other.

Hall effect measurements provide a more straightforward means of measuring the mobilities of the ionic charge carriers in NaCl and other ionic conductors and are limited only by the smallness of the effect.

The ionic Hall effect arises from the action of the magnetic field on the asymmetry in the motion of the ionic charge carriers set up by the applied electric field. It can be shown[6] that a nonzero net transverse jump probability exists, and that the temperature dependence of the Hall mobility has the same form as that of the drift mobility. The relationship between the measured rms Hall voltage V_H and the Hall mobility μ_H, in the case where ac applied fields and a square sample are used, is

$$\mu_H = 2.10\, V_H/VB \text{ (meter}^2\text{/volt-sec)},$$

where the correction for the end effect[7] has been included. Here, V is the rms voltage of the applied electric field and B is the rms flux density of the magnetic field.

We used the ac cross-modulation method[8] to make Hall measurements in Harshaw NaCl. The electric and magnetic fields were applied at 85 and 60 cps, respectively, and the 25-cps Hall voltage difference frequency was measured. Square samples and the usual four-electrode configuration were used. The Hall voltage measurement system was able to measure signals as low as 0.05 μv. The linearity of this system was assured by the use of ultralinear cathode follower[9] input stages. Elaborate precautions were taken to insure that no spurious 25-cps signals were produced in the equipment. Also, the environment of the sample was maintained so that no electronic conduction was induced in the sample.

In spite of the presence of a large amount of current noise in the crystal, Hall measurements were made at seven temperatures in the range 610-780°C. The observed Hall voltage was proportional to the magnitude of the applied fields and was not sensitive to the pressure applied on the electric field or Hall electrodes. The apparent Hall mobility (which is the difference between the Hall mobilities of the sodium and chlorine ions) calculated from these measurements is shown in Fig. 1.

The experimental points are fitted by the curve shown in Fig. 1. The shape of this curve is derived from the diffusion data of Laurent and Benard.[10]

In Fig. 2 we have compared our apparent Hall mobility data with extrapolations of the sodium ion drift mobility measurements of Etzel and Maurer[2] and of Bean.[3] The results of Etzel and Maurer as corrected by Lidiard[11] are also given in Fig. 2. The proportionality of the observed Hall voltage to the applied fields is exhibited by the low-field Hall data plotted in Fig. 2.

The Hall data are not accurate enough to permit resolution of the mobilities of the sodium and chlorine ions. However, by extrapolating from the low-temperature portion of the curve, it is

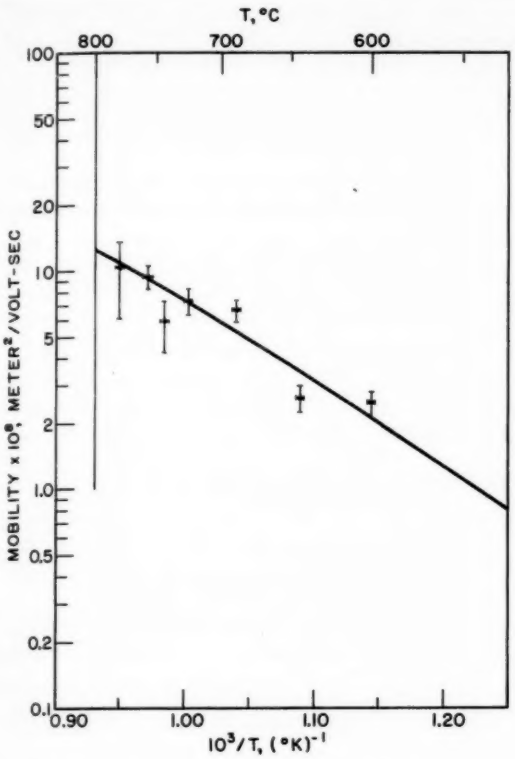

FIG. 1. Hall mobility measurements in NaCl.

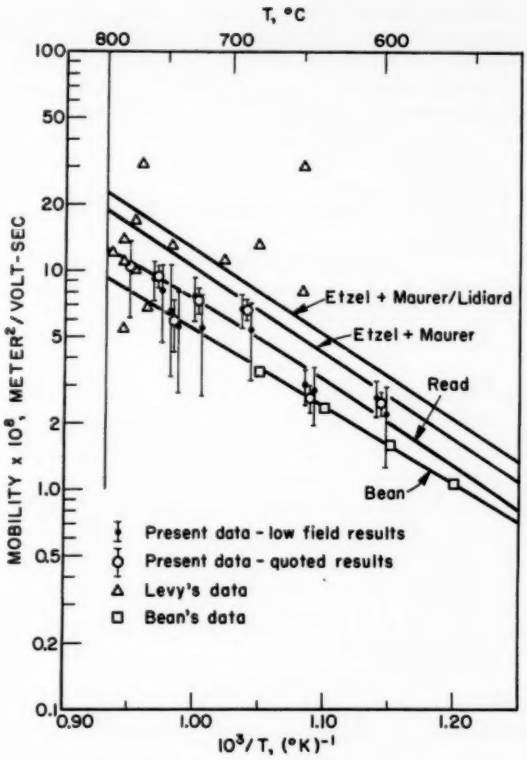

FIG. 2. Comparison of Hall data with previous mobility measurements in NaCl. The "quoted" results are identical with those of Fig. 1. They were obtained with the highest electric and magnetic fields, and are more accurate than the low-field results. The latter are included for completeness and as a check for the proper field dependences of the measured Hall voltage.

possible to make a rough estimate of the Hall mobility of the sodium ions in NaCl. The activation energy E and the pre-exponential coefficient A, defined by the equation $\mu_H = AT^{-1} \exp(-E/kT)$, are found in this way, and are given in Table I. The same type of temperature dependence is customarily applied to the analysis of drift mobility data. For comparison, the values found by the drift mobility measurements and the result of the calculation of the activation energy by Guccione et al.[12] are also given.

Table I. Values of the activation energy and the pre-exponential coefficient in the mobility expression for sodium ions in NaCl.

Source	Activation energy ev	Pre-exponential coefficient (meter)²-°K/volt-sec
Present work	0.87 ±0.10	$2.0^{+8}_{-1.5}$
Etzel and Maurer	0.85	1.96
Etzel and Maurer (Lidiard)	0.84	2.12
Bean	0.78	0.46
Guccione et al.	0.87	...

These measurements, according to our interpretation of the observed signals, represent the first observations of the ionic Hall effect in a solid. They are within the upper limits set by Levy[1] (see Fig. 2). The Hall mobility results are in the same range as the extant drift-mobility data and provide a rough verification of these previous results. In addition, the approximate equality of the Hall and drift mobilities of the ionic charge carriers in NaCl is demonstrated.

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SPIN HAMILTONIAN OF Co^{2+}

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The conventional spin Hamiltonian for an ion with $S=3/2$ at a crystalline site of cubic or tetrahedral symmetry is

$$\mathcal{H}_0 = g\beta\vec{S}\cdot\vec{H} + A\vec{S}\cdot\vec{I} - g_N\beta_N\vec{I}\cdot\vec{H}, \quad (1)$$

if the lowest orbital state of the ion is a singlet. This approximate result, derived by Abragam and Pryce,[1] has generally sufficed to describe the results of experiment. Symmetry, however, permits additional terms and, as Koster and Statz[2] and Bleaney[3] have pointed out, there should be situations[4] for which it is necessary to augment $\mathcal{H}_0$ with the terms

$$u\beta\{S_x{}^3H_x + S_y{}^3H_y + S_z{}^3H_z - \tfrac{1}{5}(\vec{S}\cdot\vec{H})[3S(S+1)-1]\}$$

$$+U\{S_x{}^3I_x + S_y{}^3I_y + S_z{}^3I_z - \tfrac{1}{5}(\vec{S}\cdot\vec{I})[3S(S+1)-1]\}, \quad (2)$$

where x, y, and z are the cubic axes of the crystal. In this Letter we report experimental results demonstrating the importance of these new terms for the spectrum of Co^{2+} in certain crystals. We also give an elementary derivation showing that one can predict the correct order of magnitude of the new terms from interactions with excited states.

If $g\beta\vec{S}\cdot\vec{H}$ is large compared with all other terms in the spin Hamiltonian and if $A\vec{S}\cdot\vec{I}$ is larger than the other terms involving $\vec{I}$, the energy levels to first order in A, u, and U are

$$W_{Mm} = g\beta HM + AMm - g_N\beta_N Hm$$

$$+(u\beta H + Um)\{M^3 - \tfrac{1}{5}[3S(S+1)-1]M\}p, \quad (3)$$

where M and m are the eigenvalues of the components along $\vec{H}$ of $\vec{S}$ and $\vec{I}$, respectively, and where

$$p = 1 - 5(n_1{}^2n_2{}^2 + n_2{}^2n_3{}^2 + n_3{}^2n_1{}^2). \quad (4)$$

Here n_1, n_2, and n_3 denote the direction cosines of $\vec{H}$ with respect to the cubic axes. The spin resonance transitions for $S=3/2$ are:

$M=+1/2$ to $-1/2$:

$$h\nu = [g - (9/5)up]\beta H + [A - (9/5)Up]m, \quad (5a)$$

$M=\pm 3/2$ to $\pm 1/2$:

$$h\nu = [g + (6/5)up]\beta H + [A + (6/5)Up]m. \quad (5b)$$

Cobalt was introduced into natural single crystals of CaF_2 and (cubic) ZnS and into synthetic single crystals of CaF_2, CdTe, and ZnSe by diffusion from suitable powders. Synthetic single crystals of ZnTe and ZnS doped with cobalt were also studied. The coordination about the metal ions is eightfold and cubic in CaF_2; in the other crystals it is fourfold and tetrahedral. The spin resonance spectrum of Co^{2+} consisted in each case of one fine-structure transition, which is assumed to be the $M=+1/2$ to $-1/2$ transition. This transition is unaffected by small noncubic terms in the Hamiltonian associated with crystalline strains. Such terms are presumed to broaden the $M=\pm 1/2$ to $\pm 3/2$ transitions beyond detection.

In accordance with (5a) the spectra are observed to be anisotropic: They can be described

in terms of an apparent g factor $g_a = g - (9/5)up$ and an apparent hyperfine interaction parameter $A_a = A - (9/5)Up$. The observed ratios $(g_a - g)/(g_a - g)_{max}$ and $(A_a - A)/(A_a - A)_{max}$ are plotted in Fig. 1 as a function of orientation. The excellent agreement with the curve for p indicates that the angular variation of the spectra is in accord with (5a). Confirmation that g_a and A_a are independent of magnetic field was furnished by measurements (for ZnS and ZnTe) at two different klystron frequencies, 14 and 20 kMc/sec. The value 3/2 of S for Co^{2+} in ZnS was verified by the detection of a weak line at one-third the resonant field for the $M = +1/2$ to $-1/2$ transition. This line is the $M = +3/2$ to $-3/2$ transition which becomes allowed in the presence of suitable strains.

Resonance parameters determined from (5a) are tabulated in Table I. The table is incomplete because of inadequate resolution of the hyperfine structure in some cases. The values for g show the same large departure from 2 found in previous studies of Co^{2+} in fourfold and eightfold coordination.[5] Our results for the anisotropy of the spectra explain previous observations[5] of anisotropy which were reported but not accounted for.

The ground term of the free Co^{2+} ion is $3d^7\ ^4F$. In a site of fourfold or eightfold cubic coordination the F state splits into two triplets (Γ_4, Γ_5)

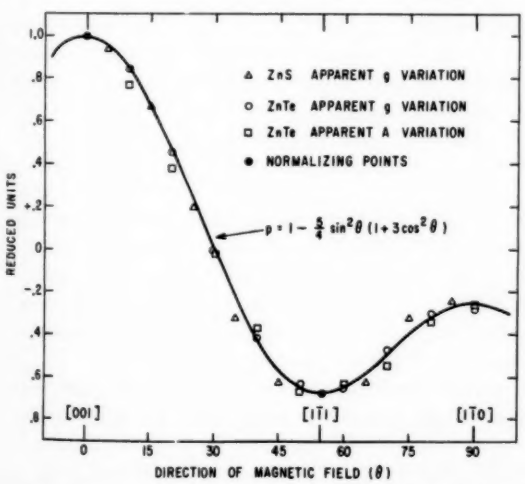

FIG. 1. Angular variation of the apparent g factor and hyperfine interaction parameter, in reduced units, for Co^{2+} in ZnS and ZnTe. The curve is the (theoretical) function p defined by (4). The magnetic field is rotated in the (110) plane.

Table I. Resonance parameters of Co^{2+} at 10°K.

Host lattice	g	u	$\lvert A\rvert$ (10^{-4} cm^{-1})	$\lvert U\rvert$ (10^{-4} cm^{-1})
ZnS	2.248	−0.0048	1.8	
ZnSe	2.270	−0.005		
ZnTe[a]	2.2972	+0.00078	17.5	0.65
CdTe[a]	2.3093	+0.0016	23.4	0.85
CaF_2	2.309	−0.009		

[a]For ZnTe and CdTe, A and U have opposite signs.

and a singlet (Γ_2), with the singlet expected to be the lowest in energy. Using fourth-order perturbation theory to include interactions[6] with the orbital triplets, we obtain from the spin-orbit interaction $\lambda(\vec{L}\cdot\vec{S})$ and the Zeeman energy term $\beta(\vec{L}\cdot\vec{H})$ or the hyperfine interaction term $P(\vec{L}\cdot\vec{I})$ the following contributions to the coefficients u and U:

$$u_1 = 120\lambda^3/(E_{52}{}^2E_{42}), \quad U_1 = Pu_1. \tag{6}$$

Here E_{52} and E_{42} denote $[E(\Gamma_5) - E(\Gamma_2)]$ and $[E(\Gamma_4) - E(\Gamma_2)]$. A second contribution to u and U from the orbital triplets arises from the spin-spin interaction[6] $-\rho(\vec{L}\cdot\vec{S})^2$ together with $\lambda(\vec{L}\cdot\vec{S})$ and $\beta(\vec{L}\cdot\vec{H})$ or $P(\vec{L}\cdot\vec{I})$ in third order. Similarly the hyperfine interaction terms $-\frac{3}{2}P\xi[(\vec{L}\cdot\vec{S})(\vec{L}\cdot\vec{I}) + (\vec{L}\cdot\vec{I})(\vec{L}\cdot\vec{S})]$ make a third contribution to U.

An estimate of u_1 and U_1 is obtained from the shift in the g factor,

$$g - 2 = -(8\lambda/E_{52}), \tag{7}$$

and the crystal field result $(E_{52}/E_{42}) = 5/9$. We obtain then from (6)

$$u_1 = -(25/192)(g-2)^3, \tag{8}$$

and from values of g given in Table I we find representative values of u_1 to be −0.0020, −0.0034, and −0.0038 for ZnS, ZnTe, and CaF_2, respectively. The corresponding values of U_1 are −0.45, −0.77, and −0.86 ($\times 10^{-4}$ cm^{-1}), using $P = +0.0225$ cm^{-1} as given by Abragam and Pryce.[7] The other contributions to u and U have been found for Co^{2+} to have the same sign as u_1 and U_1, and, when combined, have roughly the same magnitude.

While the resulting theoretical estimates of u and U agree in order of magnitude with the observed coefficients, there is no quantitative agreement. Indeed, for ZnTe and CdTe the observed values of u are considerably smaller

than the theoretical values and are of the opposite sign. However, we have considered neither perturbations from higher terms of the $3d^7$ configuration or from other excited states, nor mixing between the Γ_4 states of ^4F and those of ^4P.[7] Both effects can change the calculated coefficients significantly. Study of these questions is in progress.

The authors are indebted to the following for making crystals available: R. N. Hall (Co-doped ZnTe); A. C. Beer, Battelle Memorial Institute, (CdTe); M. H. Aven (ZnSe); A. Addamiano, (Co-doped ZnS). We also thank M. H. Aven, W. W. Piper, and J. S. Prener for furnishing Co-doped powders and for helpful discussions.

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EXPERIMENTAL VERIFICATION OF RADIATION OF PLASMA OSCILLATIONS IN THIN SILVER FILMS*

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(Received October 10, 1960)

About two years ago Ferrell[1] predicted that plasma oscillations in a thin metal film of suitable thickness should give off a photon radiation of plasma frequency ω_p. The detection and investigation of this radiation would give experimental evidence of plasma oscillations and furthermore would be a useful tool for measuring the energy of the plasmon more precisely than is possible by the electron energy-loss experiments.[2]

Ferrell[3] suggested using silver films for detecting plasma radiation. Plasmons in silver are expected to have an energy of 3.75 ev corresponding to a wavelength of 330 mμ.[4] Therefore, for radiation experiments with silver, quartz optics can be used.

In the work reported here, self-supported silver films were bombarded with 25-kev electrons. Care was taken for the electron beam to interact only with the specimen so that all radiation found necessarily had come from the silver film. To record the radiation a small quartz spectrograph was placed on the same side of the specimen as the incident electron beam. Its resolution was found to be about 10 mμ. As the spectrograph could not be calibrated absolutely, optical filters were used to fix a point in the spectrum. The dispersion was known from calibration measurements with a helium spectrum. The total number of electrons having passed the specimen could be determined, and yielded a relative measure for the exposure of the photographic film.

The spectrum of the radiation from a silver film was found to consist of a rather sharp line at 330 ± 10 mμ and a broad continuum at the long-wavelength side of the line; for shorter wavelengths the intensity rapidly falls to zero [see Fig. 1(a)]. Similar experiments with gold films with the same exposure yield only a weak continuum decreasing with wavelength.

As an experimental test whether the radiation was really caused by plasmons, we studied the dependence of the intensity on film thickness, which, as Ferrell[1] pointed out, ought to be as shown in Fig. 1(d). The experimental results [Fig. 1(a)-(c)] clearly demonstrate that the dependence of the line intensity on thickness agrees with the theoretical prediction, showing two maxima [Fig. 1(a) and (c)], the second one much

lower than the first, and the minimum equal to zero between them [Fig. 1(b)], while the intensity of the continuum apparently shows another dependence on thickness. From this fact it follows clearly that the radiation of the line is caused by plasma oscillations, since no other known process gives this dependence on thickness. As a result it is demonstrated that plasmons really can radiate, and furthermore that in silver plasmons exist with $\hbar\omega_p = 3.75 \pm 0.1$ ev.

There should exist a corresponding peak in the electron energy-loss spectrum. In all transmission experiments on silver a low-lying loss was found at 3.4 ev.[2,5] This value lies beyond the limit of error of the radiation experiment. Therefore the energy-loss spectrum was measured with the same silver films as used for detecting the radiation.[6] As Fig. 2 shows, the shape and width of the low-lying loss line agrees with that of the radiation line. The value of this loss altered a little from one measurement to the next, always lying between 3.4 and 3.6 ev. The difference between the results of the energy-loss and radiation experiments can be explained by the fact that the loss is superimposed on the slope of the no-loss peak. Therefore the maximum of the loss appears shifted towards the top of the zero peak and the measured value is too low. An estimation of the shift showed the right order of magnitude (some 0.1 ev). Probably this fact also caused the measured loss value to fluctuate. As a conclusion the radiation experiment already now yields a better, more reliable value of the plasmon energy than the energy-loss experiments, in spite of the hitherto rather rough method of wavelength determination.

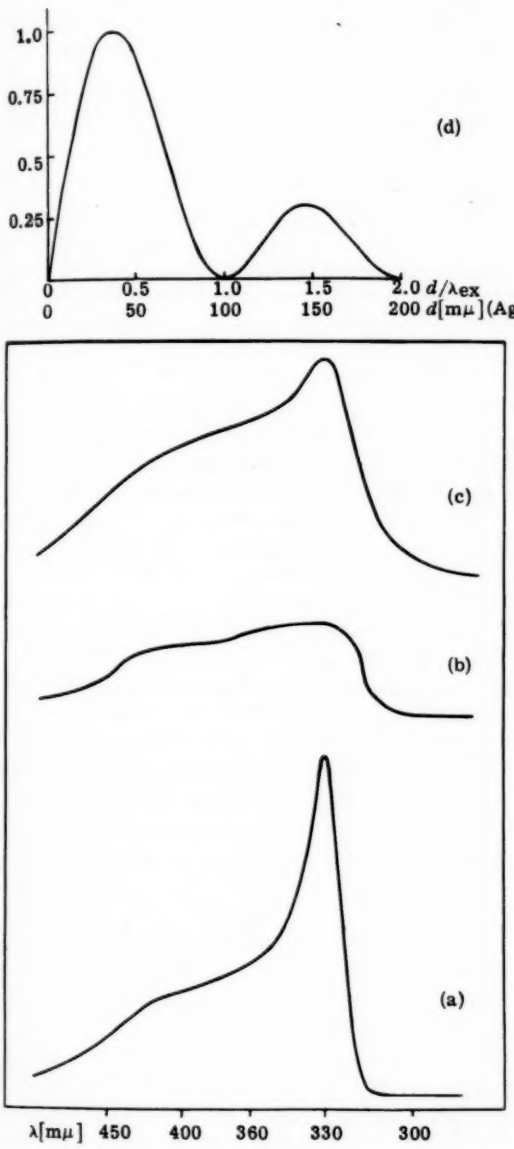

FIG. 1. Photometer curves of 3 spectra of the radiation from silver films of various thickness d: (a) $d = 45$ mμ; (b) $d = 85$ mμ; (c) $d = 150$ mμ. (d) Photon yield (in arbitrary units) per incident electron, Y, vs d/λ_{ex}. $\lambda_{ex} = \lambda_p v/c$; λ_p = plasmon wavelength; v = velocity of the incident electrons. Lower scale of (d) is specified for Ag films bombarded by 25-kev electrons.

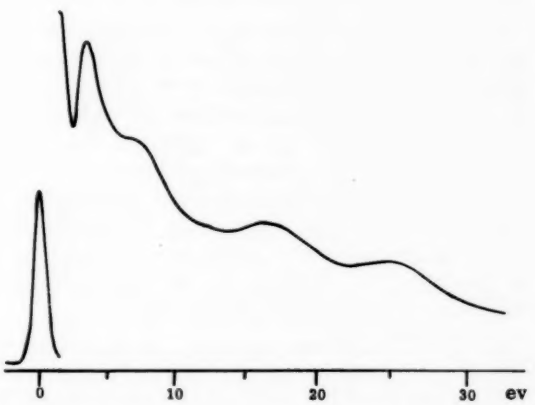

FIG. 2. Electron energy loss spectrum of a silver film ($d = 45$ mμ) (photometer curve). The no-loss peak was shielded during most of the exposure of the spectrum. Thereby its shape was preserved and the zero point of the loss axis could be fixed precisely. The losses of this spectrum are 3.59 ± 0.03, 7.4 ± 0.2, 17.0 ± 0.5, and 25.5 ± 0.5 ev.

The author wishes to express his thanks to Professor Ferrell for helpful advice and important suggestions, and to Professor Rollwagen, director of this institute, for his constant interest in this work. He is also indebted to Mr. Fischer and Mr. Otto for their performance of the energy-loss experiments.

*A more detailed report of this work will be submitted to the Zeitschrift für Physik.

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PLASMON RERADIATION FROM SILVER FILMS

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A straightforward application of the free electron theory of metals would indicate the possibility of a plasma oscillation of the valence electrons at a frequency dependent on their density. It has been conjectured[1] that the characteristic losses of energy suffered by high-energy electrons traversing thin metallic films can be attributed to the excitation of these plasmons with a characteristic frequency determined through $\Delta E = \hbar\omega_p$.

However, this explanation of the losses is not universally accepted.[2] For this and other reasons stated in his paper, Ferrell[3] has suggested a crucial test of the plasmon theory. He states that, at certain ratios of film thickness to incident electron velocity, plasmons would be excited which could be expected to decay through photon emission at a frequency ω_p. Although the level of this radiation is expected to be small, it differs from anticipated background in its dependence of intensity on electron velocity and its angular distribution about the normal to the film. The width of the line should also depend strongly on angle of observation, through the relation

$$\tau_\gamma^{-1} \propto \sin\theta \tan\theta, \qquad (1)$$

where θ is the angle from the normal and τ_γ the radiative mean life.

Although we originally planned to work with sodium films, we found it impossible to keep them continuous and clean in our apparatus. We now are working with thin silver films, in which the predictions of the free electron theory are less clear-cut but the experiment itself more simple. We deposit films of the order of 500 A thick and transfer them to cover a slit 1 mm wide at the target end of an electron gun structure providing 22-kev electrons normally incident on the film. The entire gun structure is capable of rotation about a slit serving as the input to a diffraction spectrometer which, with the exception of input lenses and a quartz window, is located entirely outside the vacuum system. The signal is taken from a photomultiplier and read beyond an amplifier and phase detector, which eliminates noise having no relation to the modulated beam.

As shown in Fig. 1(a), we find a peak in the spectrum taken at 30° from the film normal; it has

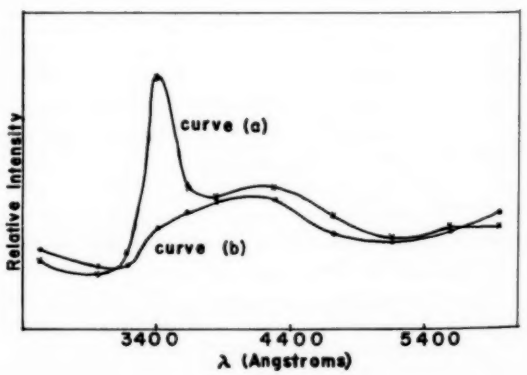

FIG. 1. Intensity of radiation vs wavelength at two different angles from normal of silver film. Thickness of film 500 A; beam voltage 22 kv. (a) $\theta = 20°$. (b) $\theta = 75°$.

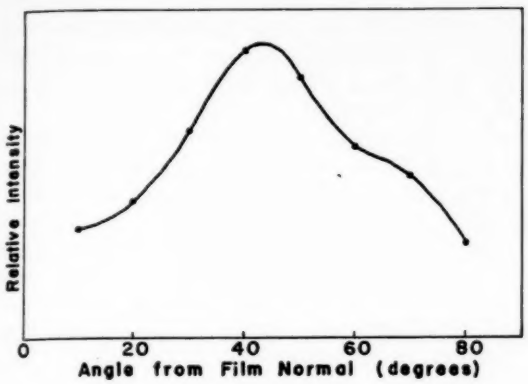

FIG. 2. Angular dependence of radiation at 3400 A under identical conditions of voltage and film thickness to those of Fig. 1.

disappeared into the background of the spectrum taken at 70° [Fig. 1(b)]. We attribute its disappearance to the expected broadening via shorter lifetime [Eq. (1)] and the expected angular dependence (Fig. 3 of reference 3). The peak in the spectrum occurs at 3400 A and coincides with the 3.7-ev loss in silver[4] and the well-known optical transparency in that metal.

An additional indication that we are indeed observing plasmon decay is provided by the angular distribution shown in Fig. 2, taken at 3400 A. This peaks around 40° to the normal to the film and appears to go to zero both at the normal and in the plane of the film, although we do not have access to those points with our equipment. This behavior is to be compared with Fig. 3 of reference 3. Comparison runs on the low-level background from gold have indicated an isotropic angular pattern.

We are attempting polarization measurements of the 3400A line from silver, but have no definitive results as yet.

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ADDITIONAL SPIN RESONANCE SPECTRUM IN ANTIMONY-DOPED GERMANIUM

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Pontinen and Sanders[1] have recently reported a spin-resonance spectrum for antimony-doped germanium which exhibits a striking new feature. In addition to the resonance of electrons bound to antimony donors which was observed by Feher, Wilson, and Gere,[2] they observe a set of four lines each of which apparently represents a g tensor of the type observed by Wilson and Feher[3] for electrons bound to donors in germanium subjected to a large elastic shear strain. We believe that the additional resonance observed by Pontinen and Sanders[1] may be accounted for by the strains normally present in germanium crystals and present our reasons in this note.

An electron bound to a donor will show a resonance line of the type observed by Wilson and Feher in strained germanium if the local strain is such as to depress the energy of one of the valleys below any other valley by a separation of at least several times Δ_c. The donor wave function then is derived from the states of that valley alone, and is insensitive to additional strain.[4] All donors satisfying this condition therefore contribute to one of the four anisotropic g tensors of the type observed by Wilson and Feher. The crucial point is that the g value is stationary with respect to other strain components, and the condition is therefore satisfied for a finite range of the ratios of strain components.

In the case of an antimony donor Δ_c is very small, and the critical strain, ϵ_c, required to separate the valleys by an amount $4\Delta_c$ is correspondingly small. It can be estimated by the relation $\Xi_u \epsilon \approx 4\Delta_c$. Using recent values[5] for Δ_c for an antimony atom and for Ξ_u, it is found that $\epsilon_c \approx 3\times10^{-5}$. Even in a relatively good germanium crystal, strains of this magnitude may exist in an appreciable part of the crystal. For example, strains larger than ϵ_c are encountered up to 10^{-4} cm from a dislocation.

Thus we divide the antimony donors into three classes: (1) those subject to a strain with a

(111)(111) component greater than ϵ_c; (2) those in the vicinity of which the strain tensor is negligible; (3) other antimony atoms. We attribute the "new" resonance to donors of class (1) and the resonance of Wilson and Feher[3] to donors of class (2). The g factors of the donors of class (3) will form a continuous distribution and will not give rise to an observed line.

Our model explains the following features of the results of Pontinen and Sanders[1]: (1) The fact that the g values of the new resonance are exactly the same and the g tensors have the same shape as those found by Wilson and Feher[3] in strained germanium. (2) The absence of the new resonance in arsenic-doped germanium. Δ_c is about an order of magnitude smaller for an arsenic atom than for an antimony atom; consequently the critical strain ϵ_c is an order of magnitude larger. The relative volume of crystal in which the strain ϵ_c may be attained will therefore be at least one and more probably two or three orders of magnitude smaller in a crystal of comparable perfection. (3) The weaker temperature dependence of the intensity of the new resonance. The intensity of the resonance observed for antimony donors not subject to strain decreases rapidly with increasing temperature because the resonance is due to electrons in the singlet ground state of the donor[2] and electrons are thermally excited out of the singlet into the triplet ground state. In the donors subject to the saturation strain, the energy difference between the lowest state and the next higher state is considerably greater than the separation of the singlet and triplet of the unstrained donor.[4] Consequently, much larger temperatures are required to excite electrons out of the lowest state, in which they produce the observed resonance.

None of these features is explained by the models suggested by Pontinen and Sanders.[1]

According to our explanation of the new resonance the relative intensities are expected to be a structure-sensitive feature of the crystal. Thus we would not have anticipated the approximate constancy of the intensity ratio for the various samples studied by Pontinen and Sanders. The resolution of this point will require studies of the new resonance in crystals of controlled and known perfection.

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QUANTITATIVE EVIDENCE OF ONE-PION EXCHANGE EFFECTS IN p-p SCATTERING*

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Only a short time ago it was possible to write of the (π) meson theory of nuclear forces, "On the one hand, no one seriously doubts that the meson theory is at least qualitatively correct, but on the other hand, not a single quantity has yet (1958) been calculated and measured with sufficient accuracy to constitute a convincing confirmation of its quantitative correctness."[1] Soon after that statement was published, however, the well-known work of Moravcsik and co-workers was reported,[2] in which a new kind of phase-shift analysis was performed on the 310-Mev proton-proton scattering data. The nine lowest angular momentum phase shifts were searched on: The higher angular momentum phase shifts were computed from a meson-theoretical one-quantum exchange force. The theoretical quantum-nucleon coupling constant was treated as another search parameter, while the theoretical virtual quantum mass was taken to be that of the physical π meson. Since the numerical value of the quantum-nucleon coupling constant turned out to be in the neighborhood of values obtained from analysis of experimental pion-nucleon data, also using meson theory, it was strong quantitative evidence for the theory.

The present author felt that the significance of the results in the work just described was considerably marred by the appearance of two strikingly different low angular-momentum phase-shift sets, both of which have approximately the same desirable behavior[3] under variations of the

theoretical coupling constant. It seemed advisable to analyze the data further, now treating the virtual quantum mass as an additional search parameter. It was assumed that if one of the sets was correct, only that one would show a behavior similar under quantum-mass variation to the behavior observed for both sets under variations in the coupling constant. This would clear away the doubt in the coupling constant prediction as well as provide a prediction for the virtual quantum mass. The latter, when compared with the well-known physically-measured mass of the pion, would provide another and perhaps more direct piece of quantitative evidence for this application of the theory.

The coupling constant was fixed at $g^2 = 12.0$, and 9-parameter searches were carried out, starting with sets 1 and 2 of MacGregor et al.[2] Contours of the goodness-of-fit parameter[2] χ^2 could have been determined in the g^2-quantum mass plane, but the profiles seemed sufficient in view of the results shown in Fig. 1. The two solutions exhibit an exceedingly poor behavior under variations of the quantum mass: The curves are far from parabolic and the minima are far from the physical value.

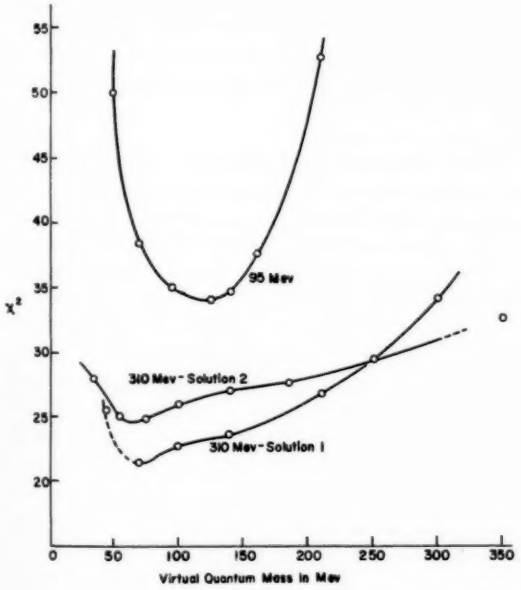

FIG. 1. The "goodness-of-fit" parameter χ^2 vs the virtual quantum mass, for the Harvard 95-Mev data and for sets 1 and 2 of MacGregor et al.[2] at 310 Mev. The 95-Mev minimum is at quantum mass ~125 Mev. The mass of the pion is ~135 Mev.

It has been recently reported[4] that a search of the 95-Mev Harvard data[5] has yielded a unique set of phase shifts. In an attempt to clarify the situation at 310 Mev, a new analysis was made of the 95-Mev data. The result of varying the coupling constant and virtual quantum mass, using a 5-parameter search, is shown in Figs. 1 and 2. The results at 95 Mev and 310 Mev are seen to be strikingly different.

The similar but poor behavior of the two 310-Mev sets in contrast to that of the 95-Mev set suggests that neither 310-Mev set is much more nearly correct than the other. That is, better 310-Mev experimental[6] data might be expected not to lead to one or the other of these, but rather to a third unique set which would have a theoretical behavior similar to that of the 95-Mev data.

We conclude that (a) the 95-Mev Harvard proton-proton scattering data sharply exclude the numerical values of the theoretical coupling constant and virtual quantum mass from being very different from the corresponding values for the real pion, thus providing quantitative evidence of the one-pion exchange force used by MacGregor et al.[2]; and (b) conclusions previously deduced from the 310-Mev data are made uncertain, probably due basically to a lack of good enough data.

The phase-shift searches reported here were

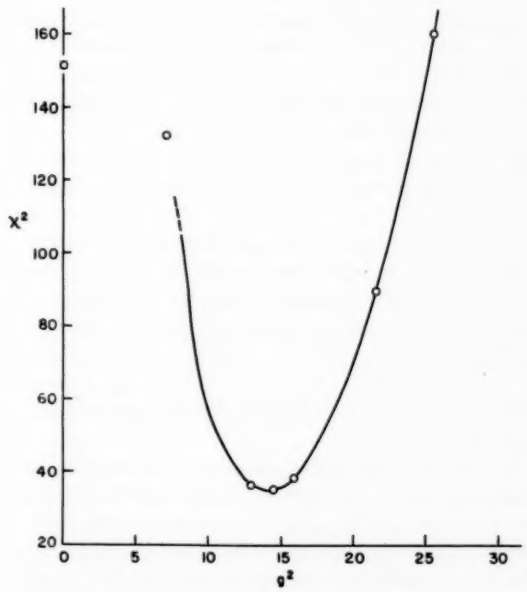

FIG. 2. χ^2 vs the quantum-nucleon coupling constant, for the Harvard 95-Mev data. The minimum is at $g^2 \sim 14$.

carried out on the IBM 704 in the U. S. Atomic Energy Commission Computation Center at New York University.

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[6]The lower values of χ^2 for the 310-Mev sets are caused in part by the large errors assigned to the experimental data at that energy.

POSSIBLE EXISTENCE OF $\Delta Q = -\Delta S$ DECAYS

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The possible existence of isotopic spin $I=3/2$ strangeness-changing currents, which lead to decays that satisfy $|\Delta S| \leq 1$ and to the $|\Delta I| = \frac{1}{2}$ rule for processes involving only strongly interacting particles, has been discussed recently.[1-3] In particular, it has been shown[1] that the quasi-conservation of the vector part of the $I=3/2$ currents in the presence of both the strong π and K interactions of the Yukawa type requires a lower symmetry of the strong couplings (the 14-parameter group $G2$) than the $I=\frac{1}{2}$ currents (the seven-dimensional rotation group). For this and other theoretical reasons,[2,3] it is worthwhile to examine in more detail the experimental implications of such currents.

As has been noted,[1] the existence of $I=3/2$ currents of strongly interacting particles which are coupled to the leptonic currents permits, but does not demand, the presence of leptonic modes of decay with $\Delta Q = -\Delta S$ (while still preserving the rule $|\Delta S| \leq 1$ for all decays). On the other hand, for pure $I=\frac{1}{2}$ currents, these decays are forbidden.

It follows from the existence of the K_{l2} decay that $I=\frac{1}{2}$ currents must be present in the space-time vector and/or axial vector part. For example, if the K were pseudoscalar (scalar), K_{l2} could occur through the axial vector (vector) part. However, this still allows one of the space-time parts to be composed of dominant, or even possibly "pure" $I=3/2$ currents through which the K_{l3} modes of decay could proceed. As the predictions of pure $I=\frac{1}{2}$ currents on the K_{l3} decays have been already given,[4] we will now examine the other "pure" case; namely, the possibility that the space-time part governing these decays consists of pure or dominant $I=3/2$ currents. It is the purpose of this note to discuss the implications of such an assumption with regard to the $\Delta Q = -\Delta S$ modes of decay. We shall also assume time-reversal invariance.

Let us consider the K_{l3} decays. The first interesting consequence is that we can obtain exact lower and upper bounds for the $\Delta Q = -\Delta S$ decays $K^0 \to \pi^+ + l^- + \nu$ in terms of observed lifetimes. For "pure" $I=3/2$ currents, neglecting electromagnetic effects, we obtain

$$\sqrt{2}\,M(K^0 \to \pi^- + l^+ + \nu) = -M(K^+ \to \pi^0 + l^+ + \nu). \quad (1)$$

By using Eq. (1) and the relation $K_2{}^0 = (K^0 - \bar{K}^0)/\sqrt{2}$, we get the following triangular inequalities for the total rates R:

$$R^{1/2}(\bar{K}^0 \to \pi^- + l^+ + \nu) + \frac{1}{\sqrt{2}} R^{1/2}(K^+ \to \pi^0 + l^+ + \nu) \geq \sqrt{2}\,R^{1/2}(K_2{}^0 \to \pi^- + l^+ + \nu), \quad (2)$$

and cyclically. These inequalities hold exactly for the e^+ and μ^+ modes and for the total rates for positive leptons, electron plus muon. The TCP theorem tells us that $R(\bar{K}^0 \to \pi^- + l^+ + \nu) = R(K^0 \to \pi^+ + l^- + \nu)$ and CP invariance implies

that the l^+ and l^- modes of $(K_2^0)_{l3}$ decay occur at the same total rate. Inserting the existing experimental data,[5,6] we obtain for the total decay rates for the leptonic modes, e plus μ:

$$8.5^{+2.3}_{-1.6} \geq \frac{R(K^0 \to \pi^+ + \text{leptons})}{R(K^0 \to \pi^- + \text{leptons})} \geq 0.85^{+0.81}_{-0.45}. \quad (3)$$

In Eq. (3), we have neglected the error in the $(K^+)_{l3}$ decays and have not corrected for possible τ^0 decays (which in this experiment[5] occur with a frequency $\lesssim 15\%$). Although the experimental errors are large, we see that the results (3) favor the existence of $\Delta Q = -\Delta S$ decays at an appreciable rate, if the assumption that these decays are governed by dominant $I = 3/2$ currents is correct.

In order to obtain more detailed predictions for the K_1^0 decays, we express K^0 in Eq. (1) in terms of K_1^0 and K_2^0 and observe that T invariance implies that the phases for the K_{l3} decays may be so chosen that all the matrix elements are real (assuming, of course, that the spin of the K is zero). If we limit ourselves to the electron modes and neglect terms proportional to m_e, each decay amplitude consists of one structure function multiplying a lepton matrix element. By taking this fact into account, the following rigorous equality follows from Eq. (1):

$$\sigma^{1/2}(K_1^0 \to \pi^- + e^+ + \nu) = |\sigma^{1/2}(K_2^0 \to \pi^- + e^+ + \nu) \pm \sigma^{1/2}(K^+ \to \pi^0 + e^+ + \nu)|, \quad (4)$$

where σ is the differential decay rate at fixed pion energy E_π. [Equation (4) does not follow necessarily for the μ modes unless one adds the dynamical assumption that the ratio of the two structure functions describing those processes is the same for the three decays.] To obtain equalities for the total rates of the electron modes, it is necessary to add the dynamical assumption that the form factor for the $(K_2^0)_{e3}$ and $(K^+)_{e3}$ decays have approximately the same dependence on E_π. This is not unreasonable since most calculations suggest that the form factor is, anyway, a slowly varying function of E_π. If this is true, Eq. (4) is also valid for the total rates and the following relations are readily obtained:

$$r \equiv R(K_1^0 \to \pi^\mp + e^\pm + \nu)/R(K_2^0 \to \pi^\mp + e^\pm + \nu) = (1+c)^2, \quad (5a)$$

$$s \equiv R(K^0 \to \pi^+ + e^- + \bar{\nu})/R(K^0 \to \pi^- + e^+ + \nu) = (1+2/c)^2, \quad (5b)$$

$$R(K^+ \to \pi^0 + e^+ + \nu)/R(K_2^0 \to \pi^- + e^+ + \nu) = c^2. \quad (5c)$$

As the experimental data include both leptonic decay modes of $(K_2^0)_{l3}$, we cannot use them directly in Eqs. (5). However, if it turns out that both modes of $(K_2^0)_{l3}$ occur at approximately the same rate (as is the case for K^+), the present experimental value for c^2 would be $1.08^{+0.43}_{-0.32}$. From this value, one obtains the predictions

$$r_+ = 4.2^{+0.8}_{-0.7}, \quad r_- = 0.00^{+0.05}, \quad (6a)$$

$$s_+ = 8.5^{+2.3}_{-1.6}, \quad s_- = 0.85^{+0.81}_{-0.45}, \quad (6b)$$

where the plus and minus subscripts refer to the two possible signs of c. This is to be compared with $r = 1$ and $s = 0$ when the $\Delta Q = -\Delta S$ decays are not present. The value r_+ is in close agreement with existing data,[7] but the experimental errors are too large at present to draw any definite conclusions. It should be noted that for pure $I = \frac{1}{2}$ currents, one has the prediction $c^2 = 1$. This result is also compatible with pure $I = 3/2$ currents if the $\Delta Q = -\Delta S$ decays occur at the rate predicted by Eqs. (5a) and (5b). Thus, an experimental determination of the ratio (5c) is not sufficient to distinguish between the $I = 1/2$ and $3/2$ currents.

It might be interesting to consider the predictions which arise in the case for which the space-time vector part of the currents is predominantly $I = 3/2$ and is quasi-conserved in the presence of both π and K interactions of the Yukawa type.[1] This requires that the K be pseudoscalar with respect to ΣN and $\Xi\Sigma$. In this case, besides the predictions of Eqs. (6), it is possible to evaluate approximately the vector contribution to $\Sigma^+ \to n + e^+ + \nu$ from the known rates of K^+ and K_2^0, if one neglects renormalization effects originating from the mass differences. The vector contribution provides a lower bound for the total rate when the electron mass is neglected. In this way, one finds that the branching ratio for $\Sigma^+ \to n + e^+ + \nu$ must be greater than $(7.3^{+2.0}_{-1.4}) \times 10^{-4}$ or $(0.73^{+0.70}_{-0.39}) \times 10^{-4}$, corresponding, respectively, to the predictions r_+ and r_- of Eqs. (6).

From the above considerations, it is clear that the search for the $\Delta Q = -\Delta S$ decays, as well as the precise determination of the K_2^0 leptonic rates, are essential in order to establish the isotopic spin structure of the strangeness nonconserving currents. If the $\Delta Q = -\Delta S$ decays are found, it would be interesting to test the numerical predictions given above to determine whether the assumption of a dominant $I = 3/2$ current re-

sponsible for the K_{l3} decays is tenable. If this were the case, the possibility that the vector current is quasi-conserved in the presence of both π and K interactions would be particularly appealing. On the other hand, if the $\Delta Q = -\Delta S$ decays do not exist and the other predictions of the pure $I = \frac{1}{2}$ hypothesis are established, it might be interesting to consider the possibility that the vector part of such currents is quasi-conserved only in the presence of the π-baryon interactions of the Yukawa type. In fact, such a restricted conservation requirement implies the absence of $I = 3/2$ currents.[1]

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REACTION $\mu + N \to e + N'$: INTERMEDIATE BOSON THEORY*

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A μ^- bound in a 1S state around a nucleus may decay via the process $\mu \to e + \nu + \bar{\nu}$ or may be absorbed by one of the nuclear protons to form a neutron plus a neutrino. However, if there exists a mechanism giving rise to a nonvanishing μ-e-γ matrix element, then one would expect a third process $\mu + N \to e + N'$ to occur. The μ^- may absorb a virtual photon from the Coulomb field of the original nucleus N, producing an electron and a recoil nucleus N' having equal and opposite momenta of magnitude 106 Mev/c. Because the photon involved is virtual rather than real, the experimental absence of $\mu \to e + \gamma$ decay does not necessarily preclude the existence of the process $\mu + N \to e + N'$. In fact, it is possible that instances of such a reaction have been observed by Sard et al., who set an upper limit 4×10^{-6} upon the branching ratio $\omega(\mu + N \to e + N')/\omega_{\text{abs}}$, although Conversi et al. have established a comparable upper limit 4.3×10^{-6} upon the branching ratio without having seen any $\mu + N \to e + N'$ events.[1] Because simple arguments allow one to suppose that the branching ratio predicted by an intermediate Boson theory of weak interactions will be close to the upper limit established by these experiments, it is appropriate at this time to investigate in detail the dependence of the predicted branching ratio upon the value of the high-momentum cutoff and the Boson mass. Furthermore, the calculation will be of value in predicting the rates of other processes like $\mu \to 3e$, which can also proceed through a μ-e-γ vertex.

A nonvanishing μ-e-γ matrix element could arise via a charged vector Boson of reasonably large mass M coupled to Fermion pairs with a coupling constant g.[2] The induced four-Fermion interaction would have the right strength, $G \approx 10^{-5}/M_{\text{nucleon}}^2$, providing g is chosen so that

$$(8)^{1/2} G = (g/M)^2. \tag{1}$$

The three Feynman graphs which contribute to the μ-e-γ matrix element in the lowest order of perturbation theory are shown in Fig. 1. Denoting the muon momentum by $p_{\mu\lambda}$, the electron momentum by $p_{e\lambda}$, and the momentum transfer by

$$q_\lambda = p_{e\lambda} - p_{\mu\lambda}, \tag{2}$$

the three contributions to the matrix element may be written as follows:

$$\langle e | J_\lambda^{\ 1} | \mu \rangle = -(2\pi)^{-7} e g^2 \bar{u}_e \tfrac{1}{2}(1-\gamma_5) \int d^4k \left[\delta_{\alpha\beta}(p_e + p_\mu - 2k)_\lambda - (p_\mu - k)_\alpha \delta_{\beta\lambda} - (p_e - k)_\beta \delta_{\alpha\lambda} - (1+\mu)(q_\alpha \delta_{\beta\lambda} - q_\beta \delta_{\alpha\lambda})\right]$$

$$\times \left(\frac{\gamma_\beta + M^{-2}(p_e - k)_\beta (\gamma \cdot p_e - \gamma \cdot k)}{(p_e - k)^2 + M^2} \right) \frac{\gamma \cdot k}{k^2} \left(\frac{\gamma_\alpha + M^{-2}(\gamma \cdot p_\mu - \gamma \cdot k)(p_\mu - k)_\alpha}{(p_\mu - k)^2 + M^2} \right) u_\mu, \tag{3}$$

$$\langle e|J_\lambda{}^2|\mu\rangle = (2\pi)^{-7} ieg^2 \bar{u}_e \tfrac{1}{2}(1-\gamma_5) K(p_e) \frac{i\gamma\cdot p_e - m_\mu}{p_e{}^2 + m_\mu{}^2} \gamma_\lambda u_\mu,$$

$$\langle e|J_\lambda{}^3|\mu\rangle = (2\pi)^{-7} ieg^2 \bar{u}_e \gamma_\lambda \frac{i\gamma\cdot p_\mu - m_e}{p_\mu{}^2 + m_e{}^2} K(p_\mu) \tfrac{1}{2}(1+\gamma_5) u_\mu. \qquad (3)$$

Here μ is the anomalous magnetic moment of the Boson in Boson magnetons, and

$$K(p) = \int d^4k \, \frac{-2\gamma\cdot k + M^{-2}(\gamma\cdot p - \gamma\cdot k)\gamma\cdot k(\gamma\cdot p - \gamma\cdot k)}{[(p-k)^2 + M^2]k^2}.$$

Neglecting the electron mass m_e and assuming that $(m_\mu/M)^2 \ll 1$ and that $-q^2/M^2 \ll 1$, the above integrals may be evaluated by expanding the denominators in powers of p_μ and p_e. Although individually the contributions are not gauge invariant, the sum $\langle e|J_\lambda|\mu\rangle$ is, and it may be written in the form

$$\langle e|J_\lambda|\mu\rangle = -ie(2\pi)^{-3}\bar{u}_e [f_0(q^2)(1-\gamma_5)(\gamma_\lambda + iq_\lambda m_\mu/q^2) + f_1(q^2)(1-\gamma_5)\sigma_{\lambda\eta}q^\eta/m]u_\mu, \qquad (4)$$

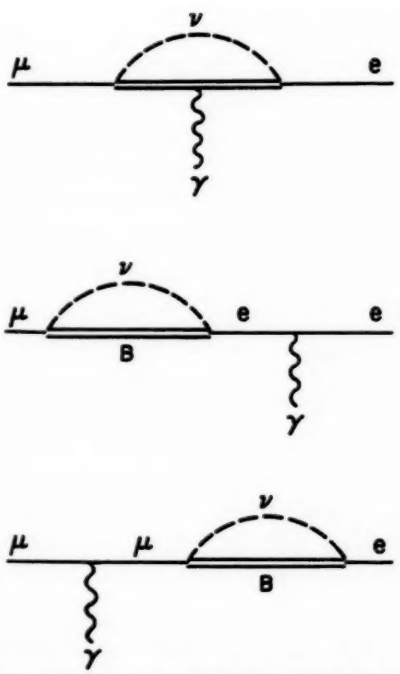

FIG. 1. Feynman graphs which contribute to the μ-e-γ matrix element. The intermediate vector Boson is designated by a double line.

where $f_0(q^2)$ and $f_1(q^2)$ are form factors, which for $-q^2 \ll M^2$ are given by

$$f_0(q^2) = -2^{-9/2}\pi^{-2} G q^2 N',$$
$$f_1(q^2) = -2^{-9/2}\pi^{-2} G m_\mu{}^2 N. \qquad (5)$$

The dependence of the dimensionless constant N upon the ratio of cutoff to Boson mass Λ/M and upon the Boson anomalous moment μ was investigated by Ebel and Ernst.[3] To study processes in which virtual photons take part it will be necessary to include N' as well as N. For these constants we find

$$N = (\mu - 1)I_0 + (2\mu + 1)I_1 - 3I_2,$$
$$N' = -\tfrac{1}{2}\mu I_{-1} - (\tfrac{3}{2}\mu + \tfrac{5}{3})I_0 + (2\mu + \tfrac{2}{3})I_1 + I_2, \qquad (6)$$

where

$$I_n = \frac{M^{2n}}{\pi^2 i}\int \frac{d^4p}{(p^2 + M^2)^{2+n}}. \qquad (7)$$

Unless $\mu = 0$ it is necessary to cope with a quadratically divergent integral I_{-1} as well as the logarithmically divergent integral I_0 studied by Ebel and Ernst. Thus, for $\mu = 0$ it suffices to introduce a cutoff factor such as $\Lambda^2/(p^2 + \Lambda^2)$ in the definition of the integrals I_n, while for non-zero Boson anomalous moment it is necessary to use a stronger cutoff factor such as $\Lambda^4/(p^2 + \Lambda^2)^2$. In Fig. 2 are given N and N' as functions of the ratio Λ/M for two values of anomalous moment, $\mu = 0$ and 0.7 Boson magneton. The latter choice of magnetic moment was shown in reference 1 to lead to a marked suppression of the muon decay mode $\mu \to e + \gamma$, i.e., $N \approx 0$. The same conclusion is reached in the present case with the stronger cutoff factor. For any cutoff less than four or five times the Boson mass the rate for $\mu \to e + \gamma$ is compatible with the experimental branching ratio $\omega(\mu \to e + \gamma)/\omega(\mu \to e + \nu + \bar{\nu}) < 2 \times 10^{-6}$.

For a wide range of elements from copper to lead, Weinberg and Feinberg have estimated the branching ratio for the reaction $\mu + N \to e + N'$

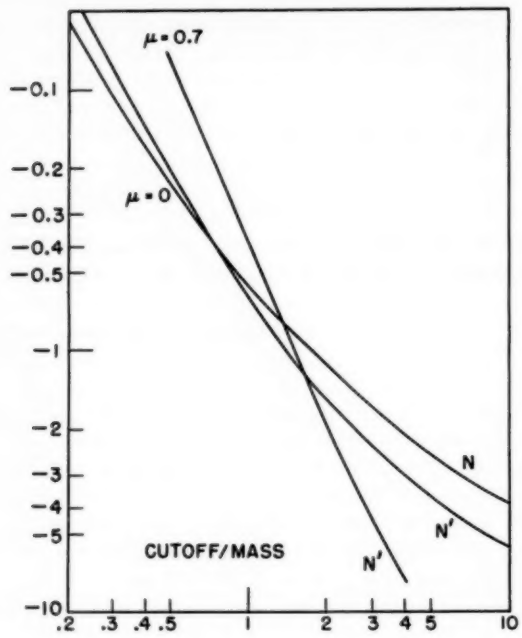

FIG. 2. Cutoff dependence of the dimensionless constants N and N' for Boson anomalous moment $\mu = 0$ and 0.7 Boson magneton. $N \approx 0$ for $\mu = 0.7$.

compared with μ absorption to be approximately[4]

$$\omega(\mu + N \to e + N')/\omega_{\text{abs}} \approx \xi_0^2/4G^2 m_\mu^4,$$

$$\xi_0^2 = |f_0(-m_\mu^2) + f_1(-m_\mu^2)|^2. \quad (8)$$

Therefore, the branching ratio according to the intermediate Boson theory is given by

$$\omega(\mu + N \to e + N')/\omega_{\text{abs}} \approx 2^{-11}\pi^{-4}(N - N')^2, \quad (9)$$

which has been tabulated for $\mu = 0.7$ (see Table I). In particular, for a cutoff equal to the Boson mass, we have

$$\omega(\mu + N \to e + N')/\omega_{\text{abs}} \approx 0.8 \times 10^{-6}. \quad [\mu = 0.7] \quad (10)$$

Table I. The branching ratio for reaction $\mu + N \to e + N'$ for various choices of cutoff Λ if the Boson anomalous moment μ is 0.7 Boson magneton. M is the Boson mass.

Λ/M	$\omega(\mu + N \to e + N')/\omega_{\text{abs}}$
0.5	$.4.1 \times 10^{-8}$
1	0.85×10^{-6}
2	2.0×10^{-5}
4	2.9×10^{-4}

On the other hand, it is to be noted that for $\mu = 0$, $N \approx N'$, and hence there is a marked suppression of the reaction $\mu + N \to e + N'$ compared with the value given in Eq. (10). Of course, the choice $\mu = 0$ gives much too large a probability for $\mu \to e + \gamma$. Therefore, if one insists that $\mu = 0$, it is necessary to impose the condition $N = 0$ and employ the intermediate Boson theory to calculate N' only. If this is done the branching ratio for $\mu = 0$ becomes (at $\Lambda = M$)

$$\omega(\mu + N \to e + N')/\omega_{\text{abs}} \approx 2 \times 10^{-6}. \quad [\mu = 0] \quad (11)$$

In conclusion, insofar as it is possible to adjust the intermediate Boson theory to account for the absence of $\mu \to e + \gamma$, this theory predicts a branching ratio $\omega(\mu + N \to e + N')/\omega_{\text{abs}}$ of order 10^{-6}, although the result is very much dependent upon the choice of cutoff.

Finally, one may use Eqs. (4) and (5) in conjunction with the graphs in Fig. 2 to evaluate the rate for any other process which goes via the μ-e-γ interaction. For example, Bander and Feinberg[5] have estimated the branching ratio for $\mu \to 3e$ compared with $\mu \to e + \nu + \bar{\nu}$ to be

$$\omega(\mu \to 3e)/\omega(\mu \to e + \nu + \bar{\nu}) \approx 6 \times 10^{-3} \xi_0^2/G^2 m_\mu^4, \quad (12)$$

where ξ_0 has been defined in connection with Eq. (8). For an anomalous moment of 0.7 Boson magneton and for a cutoff equal to the Boson mass, we have

$$\omega(\mu \to 3e)/\omega(\mu \to e + \nu + \bar{\nu}) \approx 2 \times 10^{-8}. \quad (13)$$

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PARTIALLY CONSERVED CURRENTS AND THE K' MESON*

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Very recently some theorists[1,2] have suggested that there may exist a strong S-wave $T=1/2$ K-π resonance, which can be thought of as a "particle" (called K' by Gell-Mann) similar to the K meson. Of course, there is no evidence for a second long-lived K meson, so K' must decay by strong coupling; hence $m_{K'}>m_K+m_\pi$. We are free to define the K-meson parity as odd; then the K' parity is even. There are three main reasons for suspecting the existence of such an object.

(1) It is needed to construct a highly symmetric theory[1] of the $\Delta S\neq 0$ weak interaction currents $\mathcal{J}_\lambda{}^V$, $\mathcal{J}_\lambda{}^A$.

(2) The second reason for proposing a K' again comes from the theory of the weak interactions,[3] and is related to the first. We sketch it very briefly. In our present account of the weak couplings the leptonic decays are described by at least four currents[4]; vector and axial vector with $\Delta S=0$ ($J_\lambda{}^V, J_\lambda{}^A$) and with $\Delta S=1$ ($\mathcal{J}_\lambda{}^V, \mathcal{J}_\lambda{}^A$). Hopefully, $J_\lambda{}^V$ is conserved. The divergences of $J_\lambda{}^A$ and $\mathcal{J}_\lambda{}^A$ are operators which transform like the π- and K-meson fields, respectively, and in fact one may exhibit theories in which these divergences are actually proportional to the π and K fields.[5] All this suggests that these divergences have certain "gentleness" properties,[5] properties which allow a convincing derivation[6] of the successful Goldberger-Treiman formula[7] for pion decay and of a similar formula for $K\to\mu+\nu$.

One is naturally then led to suppose also that the divergence of the $\Delta S\neq 0$ current $\mathcal{J}_\lambda{}^V$ is related to the field of a particle with the supposed properties of the K'.

(3) The well-known sharp backward peaking of the Λ^0 in the reaction $\pi^-+p\to\Lambda^0+K^0$ (in the center-of-mass system) finds an explanation if one imagines that the pole at the K' mass in the diagram of Fig. 1 dominates the associated production process.

This is in fact an old story. In the days of the parity doublet K-particle theory, several physicists[8] (e.g., Goldhaber, Schwinger) proposed a strong $KK\pi$ coupling. One of the consequences of this coupling, noted then, was just the backward peaking of Λ^0's. If there is a scalar K particle, the K', then one may have a $KK'\pi$ coupling which is perfectly parity conserving. On the basis of such a coupling, Tiomno[8] has estimated that a reasonable mass to explain the Λ^0 data would be $m_{K'}\simeq m_K+m_\pi$. Quite clearly, the K and K' are not to be taken as parity doublets in the old-fashioned sense.

A $KK'\pi$ vertex does not give rise to any distinctive new features of elastic K-nucleon scattering, but just gives additional contributions to the two-pion exchange potential, etc. Thus its effect on the KN scattering cross sections is hard to estimate, but there is no reason to suppose that it is in contradiction to anything known about them. Of course, the K' would show itself in inelastic K-nucleon scattering, through processes like $K^++p\to K'^++p$, $K'^+\to K^++\pi^0$, and this is very likely a good way to look for the K'.[1]

In this note we will consider how the K' particle may show up in the $\Delta S=1$ leptonic decay processes, assuming the divergence $\partial^\lambda \mathcal{J}_\lambda{}^V$ to be a "gentle" operator. In this case, the Goldberger-Treiman formula which comes out connects essentially unmeasurable quantities, but, as we shall see below, the argument can be turned around to yield a rather clear prediction of all features of $K_{\mu 3}$ and K_{e3} decay, except the total rate of $K_{\mu 3}+K_{e3}$.

All properties of $K_{\mu 3}$ and K_{e3} decay are determined by two scalar form factors $f_V(q^2)$ and $g_V(q^2)$, defined by the relation[7]:

$$\langle\pi|\mathcal{J}_\lambda(0)|K\rangle = \frac{i(2\pi)^{-3}}{(4E_\pi E_K)^{1/2}}\{f_V(q^2)P_{K\lambda}+g_V(q^2)q_\lambda\}, \quad (1)$$

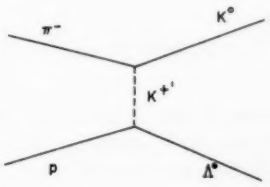

FIG. 1. Associated-production diagram with an intermediate K' particle.

where $q_\lambda \equiv (P_K - P_\pi)_\lambda$, $q^2 \equiv (\vec{P}_K - \vec{P}_\pi)^2 - (E_K - E_\pi)^2$. For example, the distribution in the pion momentum P and the angle θ between pion and neutrino is given in a system in which $P_K = (0, m_K)$, $P_\pi = (\vec{P}, E)$ by[9]

$$W(P,\theta)dPd\theta \sim (1-x^2-y^2)^2(m_K - E)^2P^2E^{-1} \times (1+x\cos\theta)^{-4}x^2|f_V(q^2)|^2 \times U(P,\theta)dPd\theta, \quad (2)$$

where

$$U(P,\theta) = \sin^2\theta + \frac{y^2}{x^2}\left|1 + \frac{m_K - E}{m_K}\frac{g_V(q^2)}{f_V(q^2)}(1+x\cos\theta)\right|^2,$$

$x = P/(m_K - E)$, $y = m_\mu/(m_K - E)$ or $m_e/(m_K - E)$, and in this frame of reference,

$$q^2 = -m_K^2 - m_\pi^2 + 2m_K E.$$

It is easy to see that the K' "pole" can appear only in $g_V(q^2)$, and not in $f_V(q^2)$. We have no idea whether this resonance gives the dominant contribution for the range of q^2 in K_{e3} and $K_{\mu 3}$ decay $[-(m_K - m_\pi)^2 \leq q^2 \leq -m_\mu^2$ or $-m_e^2]$, but to be constructive we will assume that it does. However, this is not enough to calculate g_V and f_V; we also need to know something about their behavior at high energies. We might guess, for instance, that they approach constants as $|q^2| \to \infty$, and would then need to use once-subtracted dispersion relations for them, of the form

$$f_V(q^2) = a + \int_{\mu_0^2}^\infty \frac{\rho(\mu^2)}{\mu^2 + q^2}d\mu^2,$$

$$g_V(q^2) = b + \int_{\mu_0^2}^\infty \frac{\sigma(\mu^2)}{\mu^2 + q^2}d\mu^2, \quad (3)$$

where $\mu_0 = m_K + m_\pi$. The assumption that only a sharp K-π S-wave resonance contributes to the spectral functions ρ and σ would then give

$$\rho(\mu^2) \simeq 0, \quad \sigma(\mu^2) \simeq c\delta(\mu^2 - m_{K'}^2), \quad (4)$$

so that

$$f_V(q^2) \simeq a, \quad g_V(q^2) \simeq b + \frac{c}{q^2 + m_{K'}^2}, \quad (5)$$

where c is a constant proportional to the unknown strong $KK'\pi$ coupling constant and to the amplitude for the unobservable processes $K' \to \mu + \nu$ and $K' \to e + \nu$.

So far, this is hardly a very useful set of formulas. But if we now make use of the idea that $\partial^\lambda \mathcal{J}_\lambda{}^V$ is a "gentle" operator, we can calculate not only b, but (almost paradoxically) the constant a as well. We do this by following along the lines of one of the derivations of the Goldberger-Treiman formula,[6] and assuming that the amplitude $h(q^2)$ related to $\partial^\lambda \mathcal{J}_\lambda{}^V$ by

$$\langle \pi | \partial^\lambda \mathcal{J}_\lambda{}^V(0) | K \rangle = (2\pi)^{-3}(4E_\pi E_K)^{-1/2}h(q^2) \quad (6)$$

must vanish as $|q^2| \to \infty$. Since we have

$$h(q^2) = P_K \cdot q f_V(q^2) + q^2 g_V(q^2) = q^2[\tfrac{1}{2}f_V(q^2) + g_V(q^2)] - \tfrac{1}{2}(m_K^2 - m_\pi^2)f_V(q^2), \quad (7)$$

the vanishing of $h(\infty)$ means that

$$-b = \tfrac{1}{2}a = \frac{1}{m_K^2 - m_\pi^2}\int_{\mu_0^2}^\infty [\tfrac{1}{2}\rho(\mu^2) + \sigma(\mu^2)]d\mu^2. \quad (8)$$

Assuming the dominance of the K' "pole," we have now

$$f_V(q^2) \simeq \frac{2c}{m_K^2 - m_\pi^2},$$

$$g_V(q^2) \simeq c\left[\frac{1}{q^2 + m_{K'}^2} - \frac{1}{m_K^2 - m_\pi^2}\right]. \quad (9)$$

The constancy of $f_V(q^2)$ determines the spectrum as well as the angular correlations in K_{e3} decay (since $m_e \simeq 0$, $U \simeq \sin^2\theta$) and implies that all features of $K_{\mu 3}$ decay, including the $K_{\mu 3}/K_{e3}$ branching ratio, are determined by the ratio

$$g_V(q^2)/f_V(q^2) \simeq \tfrac{1}{2}\left[\frac{m_K^2 - m_\pi^2}{q^2 + m_{K'}^2} - 1\right]. \quad (10)$$

This is a slowly decreasing function of E and of $m_{K'}$, which stays between the limits -0.04 and -0.2 in $K_{\mu 3}$ decay with the K' mass $m_{K'} \simeq m_K + m_\pi$.

It is instructive to compare our result with that obtained by Weinberg et al.,[9] who assumed that $\mathcal{J}_\lambda{}^V$ was actually conserved. In that paper

$$g_V/f_V = \tfrac{1}{2}(m_K^2 - m_\pi^2 - q^2)/q^2 \quad (11)$$

has an apparent pole at $q^2 = 0$. This is somewhat unphysical and reflects on the conserved-current hypothesis made there.[10] Their result is the same as ours would be for $m_{K'} = 0$, but in our case their unphysical pole at $q^2 = 0$ has migrated to $q^2 = -m_{K'}^2$, and represents a physical inter-

mediate state.

We will defer a detailed analysis of the consequences of Eq. (9). It should be kept in mind that we have been very optimistic in assuming that the K' resonance dominates all dispersion integrals, and that the K' lifetime (which must be of order 10^{-23} sec) is long enough for this resonance to be represented by a pole; if our optimism is justified the experiments now being planned or completed[11] on the $K_{\mu 3}$ decay should show it up. Of course, our approach in deriving Eq. (8) does not rest on any approximation but on a fundamental assumption, and so it may have a wider utility than Eq. (9).

A similar approach can be applied to hyperon beta decay, but the results obtained are less complete and very difficult to check experimentally.

If the $K'YN$ relative parity is even, then

$$\langle N|\mathcal{J}_\lambda^{\ V}|Y\rangle \sim \gamma_\chi f_V(q^2)+\sigma_{\lambda\eta}q^\eta g_V(q^2)+iq_\lambda h_V(q^2), \quad (12)$$

and our analysis gives

$$f_V(q^2)\simeq c_Y/(m_Y-m_N),\quad h_V(q^2)\simeq c_Y/(q^2+m_{K'}{}^2). \quad (13)$$

If the $K'YN$ relative parity is odd, then

$$\langle N|\mathcal{J}_\lambda^{\ A}|Y\rangle \sim i\gamma_5\gamma_\chi f_A(q^2)+q_\lambda\gamma_5 g_A(q^2) +i\sigma_{\lambda\eta}\gamma_5 q^\eta h_A(q^2), \quad (14)$$

and our analysis gives just the Gamow-Teller formulas

$$f_A(q^2)\simeq c_Y/(m_Y+m_N),\quad g_A(q^2)\simeq c_Y/(q^2+m_{K'}{}^2). \quad (15)$$

(Here $q\equiv P_Y-P_N$, and c_Y is proportional to the $K'YN$ coupling constant and to the amplitude for $K'\to\mu+\nu$.)

*Work performed under the auspices of the U. S. Atomic Energy Commission.

[1]M. Gell-Mann, Proceedings of the Tenth Annual Rochester Conference on High-Energy Nuclear Physics, 1960 (to be published).

[2]J. Tiomno, Proceedings of the Tenth Annual Rochester Conference on High-Energy Nuclear Physics, 1960 (to be published).

[3]J. Bernstein and M. Gell-Mann (unpublished).

[4]For an alternate scheme see A. Pais, CERN preprint, 1960, and further unpublished work which was kindly explained to us by Professor Pais.

[5]M. Gell-Mann and M. Lévy, Nuovo cimento 16, 705 (1960).

[6]Y. Nambu, Phys. Rev. Letters 4, 380 (1960); J. Bernstein, S. Fubini, M. Gell-Mann, and W. Thirring (to be published). See also Chou Kuang-chao, Dubna report, 1960 (unpublished).

[7]M. L. Goldberger and S. B. Treiman, Phys. Rev. 110, 1178 (1958).

[8]See, e.g., J. Steinberger, Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1958 (United Nations, Geneva, 1958). A similar mechanism operates in the reaction $\pi^-+p\to K^0+\Sigma^0$, in which the Σ^0 is also observed to go backwards, and in the reaction $\pi^++p\to\Sigma^++K^+$ where the backwards peaking has not been observed, although the data here are especially meager. It is easy to see that there is no K' mechanism available to the reaction $\pi^-+p\to\Sigma^-+K^+$ since the intermediate K' would have to have an impossible charge. For this case the experiments show isotropy at lower pion energies and some forward peaking at the high energies. J. G. Taylor, Nuclear Phys. 9, 357 (1959), has noted that in the usual theories a firm establishment of this pole would very likely indicate that K^+ and K^0 have opposite parities. For the early theoretical papers see M. Goldhaber, Phys. Rev. 101, 433 (1956), and J. Schwinger, Phys. Rev. 104, 1164 (1956).

[9]See A. Pais and S. B. Treiman, Phys. Rev. 105, 1616 (1957), and S. Weinberg, R. E. Marshak, S. Okubo, E. C. G. Sudarshan, and W. B. Teutsch, Phys. Rev. Letters 1, 25 (1958). Our notation is essentially that of the latter reference. Equation (7) of this reference should have $1-x^2$ replaced by x^2-1.

[10]One of us (J.B.) has profited from several discussions with D. A. Geffen and M. Gell-Mann on this point and others related to the K' particle.

[11]D. Glaser and D. Ritson (private communications).

ABSTRACTS

In this section are printed the abstracts of Articles that have been forwarded to The American Institute of Physics for publication in THE PHYSICAL REVIEW. In quoting information obtained from this section before the appearance of the corresponding Article, reference should be made to "Physical Review (to be published)" rather than to this Journal.

SPUTTERING OF SILICON WITH A^{+2} IONS. S. P. Wolsky and E. J. Zdanuk, Research Division, Raytheon Company, Waltham, Massachusetts (Received August 26, 1960).

A gravimetric technique involving a sensitive quartz microbalance was used for the determination of sputtering yields for the argon ion-bombardment of silicon. The sputtering yield for A^{+2} ions was deduced from the results of experiments in which the relative concentrations of A^{+} and A^{+2} ions were varied in a known manner. On the assumption that sputtering is a kinetic energy controlled phenomenon, we would expect $S_E(A^{+2}) = S_{2E}(A^{+})$, where S_E is the number of atoms sputtered by an impinging ion of energy E. This investigation showed, however, that $S_E(A^{+2}) = 4S_{2E}(A^{+})$. This result indicates the influence in the sputtering process of some other factor in addition to the ion kinetic energy.

ULTRASONIC ATTENUATION IN SUPERCONDUCTORS. T. Tsuneto, Department of Physics, University of Illinois, Urbana, Illinois (Received August 31, 1960).

A general treatment of ultrasonic attenuation of both longitudinal and transverse waves in superconductors, valid for an arbitrary mean free path, is given on the basis of the Bardeen-Cooper-Schrieffer theory. The interaction between the ultrasonic waves and electrons is assumed to be given by a self-consistent electromagnetic field. Instead of the customary theory of the attenuation based on the Boltzmann equation, a different formulation is developed using the density matrix formalism. The ratio of the attenuations in superconducting and normal metals for the longitudinal wave turns out to be approximately independent of the mean free path. The attenuation of the shear wave due to electromagnetic interaction is shown to be very small in the superconducting state.

LONGITUDINAL FERRIMAGNETIC RESONANCE. Roald K. Wangsness,* U. S. Naval Ordnance Laboratory, White Oak, Maryland (Received September 2, 1960).

Susceptibility components are calculated for a triangular ferrimagnet when the oscillating field is parallel to both the constant field and the net magnetization. Two new effects are found which are analogous to that discussed previously. They consist in the production of oscillating magnetization components which are of the same frequency as the external field and which are parallel and perpendicular to the net magnetization.

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BAND STRUCTURE OF NOBLE METAL ALLOYS: OPTICAL ABSORPTION IN Cu-Ge ALLOYS AT 4.2°K. J. A. Rayne, Westinghouse Research Laboratories, Pittsburgh, Pennsylvania (Received September 12, 1960).

Calorimetric optical absorption measurements at 4.2°K have been made on a representative series of copper-germanium alloys, over the wavelength range 0.23 to 4 microns, using electropolished bulk specimens. The variation of the infrared absorptivity with residual resistance for the dilute alloys confirms that the impurity relaxation time for copper is anisotropic. Changes in the absorption spectrum below 6000 A are inconsistent with the rigid band model, but may be reconciled with the theory of Cohen and Heine. The shift in the main absorption edge is analyzed to give information about the screening of the solute atoms in these alloys.

CORRELATION EFFECTS ON THE ELECTRONIC SPECIFIC HEAT OF SODIUM. Edward A. Stern, University of Maryland, College Park, Maryland (Received June 20, 1960).

The effect of the martensitic transformation in sodium on measured values of its specific heat is analyzed. It is shown that the only important effect at low temperatures is to produce a mixture of the two phases. The measured specific heat is very closely given by the sum of the specific heats of each phase measured separately. An analysis of the various experimental measurements gives the result that the effective mass of conduction electrons in the bcc high-tempera-

ture phase is greater than 1.5 times the free electron mass while in the hcp low-temperature phase the corresponding value is less than 1.21. Using further experimental data it is estimated that the effective masses in the bcc high-temperature phase and the hcp low-temperature phase are 1.7 and 1.1 times the free-electron value, respectively. These effective mass values imply that there is substantial contact of the Fermi surface in the hcp structure with the A faces of the Brillouin zone. The large effective mass in the bcc phase indicates a large enhancement of the specific heat of the conduction electrons in sodium by correlation and electron-phonon interaction effects. The correlation effects alone appear to increase the specific heat of a free electron gas at a density corresponding to $r_s = 3.96$ by about 40%.

ORIGIN OF HYPERSONIC ATTENUATION IN GERMANIUM AT LOW TEMPERATURES. G. S. Verma and S. K. Joshi, Department of Physics, University of Allahabad, Allahabad, India (Received August 2, 1960; revised manuscript received September 13, 1960).

Hypersonic attenuation in germanium at different temperatures is calculated and compared with the recent low-temperature measurements by Truell and his associates. The excellent agreement between the theory and the experiment for the temperature dependence shows that Umklapp processes are responsible for the attenuation at low temperatures.

NATURE OF VIBRATIONAL MODES IN IONIC CRYSTALS. Herbert B. Rosenstock, U. S. Naval Research Laboratory, Washington, D. C. (Received August 23, 1960).

The principal purpose of this paper is to discuss the nature of the modes of vibration in a solid. The following is found: (1) Waves in lattices are in general neither transverse nor longitudinal; in particular, they need not be transverse or longitudinal when the propagation vector $\vec{k}$ is very small. (2) The relationship $\omega_l/\omega_t = (\epsilon_0/\epsilon_\infty)^{1/2}$ for "longitudinal" and "transverse" modes in ionic crystals applies, if at all, in a region of small, but nonzero, wave vector $\vec{k}$. (3) The derivation of this relationship is based, at least implicitly, on the use of cyclic boundary conditions. (4) The use of cyclic boundary conditions is valid in statistical problems for crystals without long-range forces, but has never been justified for systems with Coulomb forces. (5) If cyclic boundary conditions are nonetheless used, it can be simply shown that for $\vec{k} = 0$, $\omega_l/\omega_t = 1$.

FORMATION OF F CENTERS IN KCl BY IONIZING RADIATION. P. V. Mitchell,* D. A. Wiegand, and R. Smoluchowski,† Carnegie Institute of Technology, Pittsburgh, Pennsylvania (Received June 23, 1960).

Measurements have been made of the growth of the F band in undeformed, deformed, and heat-treated KCl crystals at room temperature using 140-kvp x-ray irradiation. The growth curves were analyzed in terms of the initial concentration of negative-ion vacancies in the lattice before irradiation, rate of formation of new vacancies, rate of electron capture by the initial and by the new vacancies, and bleaching constants. These parameters were studied as a function of the intensity of irradiation. It appears that the new vacancies are generated at a rate proportional to the square of the intensity of irradiation and inversely proportional to the dislocation density. Deformation increases the concentration of initial vacancies near dislocations, while heat treatment increases the concentration of initial vacancies dispersed in the volume of the crystal. Since neither the jog mechanism nor the Varley mechanism can account for the observations, a new model is proposed which accounts for all the observed phenomena.

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α AND β BANDS IN SODIUM CHLORIDE. J. D. Rigden,* Radio and Electrical Engineering Division, National Research Council, Ottawa, Canada (Received September 6, 1960).

The peak energy of the α and β bands in sodium chloride crystals have been measured at 77° with a vacuum ultraviolet monochromator. The β band was well defined, and occurred at 7.40 ev with a half-width of 0.4 ev. The oscillator strength of

the β band was calculated to be 0.55. No resolved α band could be produced, but subtraction of absorption curves indicated that it occurred near 7.20 ev.

*From November, 1960, at Bell Telephone Laboratories, Murray Hill, New Jersey.

FREQUENCY-DEPENDENT HALL EFFECT IN NORMAL AND SUPERCONDUCTING METALS. Piotr B. Miller, Department of Physics, University of Illinois, Urbana, Illinois (Received September 12, 1960).

The Hall current flow occurring in a normal and a superconducting metal when both a static magnetic field (H_0) and an electromagnetic wave are applied on the metal is calculated. The entire frequency range of the electromagnetic wave is discussed although the emphasis is on the microwave range. The nonlocal, transverse Hall current in a normal metal is calculated by solving the Boltzmann equation. It is shown that the microwave Kerr rotation in a circular cylindrical cavity provides a good test for the nonlocal Hall current in a normal metal. The relation between a longitudinal and a transverse Hall current in a superconductor is briefly discussed. A detailed theory of the transverse Hall current in a superconductor based on the Bardeen-Cooper-Schrieffer model and including the effect of collective excitations is presented. The field H_0 is assumed constant in space and a general result for the Hall current in Q space is derived. When the electric field is constant in space ($Q \to 0$), it is shown that the Hall current is proportional to the microscopic analog of the fraction of normal electrons of a two-fluid model.

INTERACTION OF A POLARIZABLE POTASSIUM CHLORIDE CRYSTAL WITH A VALENCE-BAND HOLE. Stephen J. Nettel,* Laboratory for Insulation Research and Solid-State and Molecular Theory Group, Massachusetts Institute of Technology, Cambridge, Massachusetts (Received April 15, 1960; revised manuscript received September 15, 1960).

The interaction of a valence-band hole with a potassium chloride crystal, when crystal-hole correlations are considered, is studied by variational means. Initially a crystal trial wave function is constructed which allows for the ionic polarizability of the crystal by means of a correlation between the crystal configuration and the motion of the hole. The expectation value of the Hamiltonian operator for the crystal is found by integrating over both electronic and nuclear coordinates. The necessary matrix elements of the electronic energy operators are taken from a previous calculation by Howland. The total energy expectation is minimized with respect to a single parameter in the wave function that measures the hole-lattice correlation. One finds that the valence bands obtained when the crystal lattice is treated as rigid become completely flat, a result which implies that the hole is self-trapped.

The modifications that are introduced by the addition of the electronic polarizability are studied by repeating the previous calculation with a refined wave function. Only a rough treatment drawing on the experimental electronic polarizabilities of the crystal ions is given. Information on self-trapped holes in KCl derived from the electron-spin resonance experiments of Castner and Känzig is briefly considered.

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ELECTRON SCATTERING BY THIN FOILS FOR ENERGIES BELOW 10 kev. H. Kanter, Electronics and Nuclear Physics Department, Westinghouse Research Laboratories, Pittsburgh, Pennsylvania (Received May 23, 1960; revised manuscript received September 19, 1960).

The transmission (η_T) of electrons through thin films of C, Al_2O_3, Al, Ni, Ag, and Au, together with their distribution in angle and energy, were measured in a spherical retarding potential analyzer. The distributions were characterized by average and most probable scattering angle, average and most probable fractional energy loss, etc. The dependence of these variables on initial energy (E_p), film thickness, and material was investigated. For sufficient film thickness, the transmitted energies, the scattering angles, and η_T can be represented as universal functions of the reduced energy, E_p/E_c, where E_c is the critical E_p for the onset of transmission. Direct relations exist between η_T, scattering angles, and energy losses for the complete range of scattering from small-angle scattering to total diffusion. The dependence of η_T and average fractional energy loss on Z is consistent with

published results on backscattering coefficient and energy loss for thick layers. An estimate of the mean free path for inelastic collisions proves to be in good agreement with the predictions of the Bohr-Bethe theory. Range-energy relations are almost independent of Z when the range is measured in mass per unit area.

EXCHANGE INTERACTION BETWEEN CONDUCTION ELECTRONS AND MAGNETIC SHELL ELECTRONS IN RARE-EARTH METALS. S. H. Liu, Institute for Atomic Research and Department of Physics, Iowa State University, Ames, Iowa (Received September 1, 1960).

The interaction Hamiltonian for the Coulomb exchange effect between conduction electrons and magnetic shell electrons in rare-earth metals is derived from first principles. The approximations under which the interaction can be represented by the product of electron and ion spin vectors are exhibited.

PARAMAGNETIC RESONANCE OF Fe^{3+} IN OCTAHEDRAL AND TETRAHEDRAL SITES IN YTTRIUM GALLIUM GARNET (YGaG) AND ANISOTROPY OF YTTRIUM IRON GARNET (YIG). S. Geschwind, Bell Telephone Laboratories, Murray Hill, New Jersey (Received September 14, 1960).

The electron paramagnetic resonance spectrum of a small Fe^{3+} impurity which enters substitutionally for the gallium in single crystals of yttrium gallium garnet (chemical formula $Y_3Ga_5O_{12}$) has been examined at 24 kMc/sec at 295°K and 1.6°K. Fe^{3+} is studied for the first time in tetrahedral coordination. The results for the crystal field parameters that appear in the usual spin Hamiltonian for Fe^{3+} for the octahedral (a) and tetrahedral (d) sites are, in cm^{-1}:

$$a_a = +0.0185,\ D_a = -0.1294,\ F_a = +0.0039,$$
$$a_d = +0.0062,\ D_d = -0.0880,\ F_d = -0.0026.$$

The finding of a positive in both types of sites where the cubic crystalline potential, V, has opposite signs indicates that in the mechanism responsible for this splitting, terms proportional to even powers of V are dominant. Using the experimentally determined crystal field parameters of Fe^{3+} in YGaG, the low-temperature anisotropy energy per unit cell in the isostructural ferrimagnet, YIG, is predicted as $K_1 = -0.380\ cm^{-1}$. This is 50% larger than the experimental value $K_1 = -0.250\ cm^{-1}$ and several sources for the origin of this discrepancy are suggested.

RECOMBINATION RADIATION FROM SILICON UNDER STRONG FIELD CONDITIONS. L. W. Davies* and A. R. Storm, Jr., Bell Telephone Laboratories, Murray Hill, New Jersey (Received September 6, 1960).

In an attempt to determine the distribution in energy of hot electrons and holes in silicon placed in an intense uniform electric field, measurements have been made of the spectral distribution of recombination radiation at 77°K (field strengths up to 3700 v cm^{-1}) and at 20°K. No change in the spectrum with field was observed, other than a rise in temperature of 6° at 77°K due to Joule heating at 3700 v cm^{-1} in the sample, from which it was concluded that recombination radiation at these temperatures arises predominantly from the decay of excitons formed from the hot carriers, and that the excitons have a thermal distribution of energy at the lattice temperature. In addition, results are given for the spectrum of the radiation from avalanche breakdown regions in reverse-biased silicon p-n junctions at 77° and 300°K; no differences were detected in the range of energies 1.0–1.4 ev, from which it was concluded that exciton decay does not contribute to the observed radiation at 77°K.

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ENERGY BANDS IN THE BISMUTH STRUCTURE: I. A NONELLIPSOIDAL MODEL FOR ELECTRONS IN Bi. Morrel H. Cohen,* Institute for the Study of Metals, University of Chicago, Chicago, Illinois, and Hughes Research Laboratories, Malibu, California (Received September 6, 1960).

The band structure near a minimum at a point of no special symmetry is examined for energies small compared to all band gaps except that to the next lower band. Spin-orbit coupling is included. The theory is specialized to points having the three possible symmetries of electrons in Bi and further simplifications appro-

priate to Bi are made. The resulting nonellipsoidal energy surfaces are studied in some detail. An experiment is suggested which is capable of distinguishing between the three possibilitites. Fitting the model to existing information is not carried out in this paper.

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EFFECT OF PHOTOEXCITATION ON THE MOBILITY IN PHOTOCONDUCTING INSULATORS. Richard H. Bube and Harold E. MacDonald, RCA Laboratories, Princeton, New Jersey (Received September 1, 1960).

The Hall mobility of carriers in photoconducting insulators can be varied over an appreciable range by the effects of photoexcitation. Such a variation can result either (1) from a change in the density of scattering centers as the result of a change in the occupation of imperfection centers, or (2) from the initiation of two-carrier conductivity. Suitable use of the phenomena involved in the photo-Hall effect can lead not only to knowledge about carrier density, carrier sign, and carrier mobility, but also about the charge on imperfection centers, and to an independent determination of the cross section of imperfection centers. Experiments on CdS and CdSe single crystals with conductivities lying between 10^{-9} and 10^{-1} mho/cm are described to illustrate the potentialities of the technique. The results emphasize both the importance that changes in mobility can play in normal photoconductive processes, and the importance of hole conductivity under suitable circumstances.

STARK BROADENING OF SPECTRAL LINES BY HIGH-VELOCITY CHARGED PARTICLES. M. Lewis,* Sloane Physics Laboratory, Yale University, New Haven, Connecticut (Received January 14, 1960; revised manuscript received October 3, 1960).

The broadening of the Lyman α line by high-velocity charged particles is calculated in the classical path approximation without the completed-collision assumption. For noninteracting perturbers, the divergence at large impact parameters associated with usual impact theories does not arise. Interactions between the perturbers are introduced by the pair correlation function. The resulting line shape is valid for frequencies larger than those permitted by the impact theory.

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INTERFERENCE EFFECTS IN THE RESONANCE FLUORESCENCE OF "CROSSED" EXCITED ATOMIC STATES. P. A. Franken, The Harrison M. Randall Laboratory of Physics, The University of Michigan, Ann Arbor, Michigan (Received September 12, 1960).

The recent spectroscopic method developed by Colegrove, Franken, Lewis, and Sands exploits interference effects which occur in the resonance fluorescence of atoms exhibiting pairs of "crossed" excited states. Some of the theoretical features of the technique are discussed in terms of the formalism developed by Breit from which the salient features of the observed lineshapes can be readily deduced. Alternative derivations of the Breit formula are given together with a discussion of the nature and representation of the requisite resonance radiation.

CONTINUOUS ABSORPTION DUE TO FREE-FREE TRANSITIONS IN HYDROGEN. Takashi Ohmura* and Haruko Ohmura,* Division of Pure Physics, National Research Council, Ottawa, Canada (Received August 31, 1960).

The absorption of radiation in the free-free transition of the negative hydrogen ion is the inverse process of bremsstrahlung from an electron in the vicinity of a neutral hydrogen atom. This process is the most important cause of continuous absorption by the atmosphere of the sun and stars in the infrared region. The transition matrix element is expressed, with sufficient accuracy, in terms of the s-phase shifts of electron-hydrogen atom scattering alone. To meet the requirement in astrophysical studies, the continuous absorption coefficients due to the free-free transition have been tabulated for a wide range of wavelengths (4050 A to infinity) and temperatures (2520 to 10 080°K) of the hydrogen gas, by using the s-phase shifts of e-H scattering which include exchange and correlation effects between two electrons. The tabulated coefficients are 20–70% less than the ones computed by

Chandrasekhar and Breen, who used Hartree functions without exchange. Further improvement of the present result is discussed.

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ISOTOPE SHIFTS IN THE SPECTRA OF Mo AND Ru. R. H. Hughes, Department of Physics, University of Arkansas, Fayetteville, Arkansas (Received August 29, 1960).

Isotope shifts in several lines showing shifts in the field-effect direction in the spectra of molybdenum and ruthenium have been studied with the aid of a Fabry-Perot interferometer. The variations in the shifts are quite similar in the two elements. A distinct minimum shift between the even-even nuclei occurs at neutron number 56. Extreme even-odd staggering inverts the expected order of the atomic levels belonging to nuclei with neutron numbers 54 and 55. The variations in the shifts were qualitatively predicted by the nuclear deformations as measured by Coulomb excitation, particularly in the case of molybdenum.

RECOMBINATION OF IONS AND ELECTRONS. N. D'Angelo, Project Matterhorn, Princeton University, Princeton, New Jersey (Received September 9, 1960).

A process of electron-ion recombination is considered, involving three bodies (one ion and two electrons), in which an electron, as a result of a collision with another electron, loses enough energy to be captured in one of the excited electronic orbits of the ion and then ends in the ground state by emission of one or more light quanta. It is shown that such a process might account for the large values of the recombination coefficient found experimentally.

COMPARISON OF THE REACTIONS $A^{36}(d,p)A^{37}$ AND $A^{36}(d,n)K^{37}$. S. S. Yamamoto* and F. E. Steigert, Yale University, New Haven, Connecticut (Received September 7, 1960).

The mirror reactions $A^{36}(d,p)A^{37}$ and $A^{36}(d,n)K^{37}$ have been studied at 3.85-Mev bombarding energy. In the first, Q values of 6.55, 5.16, 4.92, 3.98, and 3.00 Mev were observed. The stripping distributions may be described in terms of l_n values of 2, 0, 2, 2, and 2, respectively. In the second, Q values of -0.32 and -1.78 Mev were observed. The former followed an $l_p = 2$ angular distribution. The latter could be described by a sum of $l_p = 0$ and $l_p = 2$ distributions, suggesting an unresolved doublet.

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SLOW-NEUTRON SCATTERING CROSS SECTIONS OF TERBIUM, YTTERBIUM, AND LUTETIUM. Masao Atoji, Institute for Atomic Research and Department of Chemistry, Iowa State University, Ames, Iowa (Received July 13, 1960).

The coherent neutron scattering cross sections of terbium, ytterbium, and lutetium as determined from the neutron diffraction measurements on TbC_2, YbC_2, and Yb and Lu metals are found to be 7.2 ± 0.4, 20.0 ± 0.4, and 6.70 ± 0.37 barns, respectively, all with amplitude of positive phase. The total scattering cross sections of Tb, Yb, and Lu and the magnetic scattering cross sections of Tb^{+3} and Yb^{+3} are evaluated for thermal neutron energies. The neutron diffraction data also show the Debye temperatures of Yb and Lu metals to be $109 \pm 5°K$ and $161 \pm 7°K$, respectively.

INFLUENCE OF MAGNETIC INTERACTION ON NUCLEAR ORIENTATION OF COBALT-60 IN RARE EARTH DOUBLE NITRATES. M. W. Levi,* R. C. Sapp, and J. W. Culvahouse, Department of Physics and Astronomy, The University of Kansas, Lawrence, Kansas (Received August 5, 1960).

The effect of ionic spin-spin interactions on nuclear orientation of cobalt-60 in cerous zinc nitrate has been investigated in single-crystal spherical samples cooled below 1°K by adiabatic demagnetization. An experimental demonstration is given that Ce-Co interaction is indeed the cause of the apparently anomalous alignment data. By hypothesizing the existence of a temperature-dependent internal magnetic field, exerted on cobalt ions by spatially ordered cerium dipoles below the critical temperature, a simple but quantitatively satisfying description of the observed phenomena is obtained in the molecular field approximation with $T_C = 6.9 \pm 0.1°K$ and $H_0 = 165 \pm 20$ oersteds. The nature of the ordered state could not be inferred in detail, however.

In the theoretical calculations, measured values of g factors and hfs splittings of cobalt ions in the cerium salt are used where appropriate. Crystal structure information is invoked only for the purpose of a symmetry argument. Finally, the effectiveness of Ce-Zn nitrate as a paramagnetic medium for nuclear orientation experiments is pointed out.

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GAMMA-RAY ANISOTROPIES FROM ORIENTED Pm^{144}. D. A. Shirley, J. F. Schooley,* and J. O. Rasmussen, Lawrence Radiation Laboratory and Department of Chemistry, University of California, Berkeley, California (Received September 15, 1960).

Radioactive Pm^{144} was oriented at low temperatures in a single crystal of neodymium ethylsulfate. The gamma rays at 475, 615, and 695 kev were found to be anisotropic. The results confirm the decay scheme previously proposed as well as crystal field calculations for Pm^{+3} in this lattice. It is not possible to decide between spin 5 and spin 6 for Pm^{144}. Values were obtained for $|A|/k$ and $|\mu|$ of 0.0091°K and 1.68± 0.14 nm for $I=5$, or 0.0079°K and 1.75± 0.14 nm for $I=6$. The lowest doublet of Pm^{+3} was found to be split, presumably due to Jahn-Teller distortion.

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(d,p) REACTION ON HEAVY ELEMENTS AT LOW DEUTERON ENERGIES. Richard H. Stokes, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico (Received September 6, 1960).

Deuterons of 9.1, 8.3, and 7.4 Mev were used to produce (d,p) reactions in Pb^{206} and Bi^{209} targets. The proton differential cross section was measured for different Q values, each of which corresponds to a final state of known assignment. With one exception, all of the observed angular distributions were broad peaks with maxima near 180°. The theoretical approximations which apply for low deuteron energy predict a Gaussian distribution peaked in the backward direction. Although the measured distributions are not of Gaussian form, a comparison of the measured and predicted width variation with Q shows fair agreement with one theoretical result and poor agreement with the other. For the reaction with the highest $Q(\approx 4.5$ Mev), a peak near 120° was observed. This more forward peak would be expected both from a reduced Coulomb effect and from the influence of the nuclear potential on the proton. As expected when the Coulomb field is dominant, there was only a small observed correlation between the measured angular distribution and the angular momentum of the captured neutron. In a few cases, triton angular distributions from (d,t) reactions were measured, and these also showed peaks at large scattering angles.

DIRECTIONAL AND POLARIZATION CORRELATION STUDIES IN THE DECAY OF 5-HOUR Sb^{118}. M. K. Ramaswamy, W. L. Skeel, D. L. Hutchins, and P. S. Jastram, Physics Department, Ohio State University, Columbus, Ohio (Received September 13, 1960).

Directional and polarization correlation measurements have been carried out on the gamma-rays accompanying the decay of 5-hour Sb^{118}. These measurements lead to the following spin and parity assignments for excited levels in Sn^{118}: 1.22 Mev (2^+), 2.25 Mev (4^+), and 2.51 Mev (5^-). A search for positrons was made, and an upper limit on positron emission was set at 0.2% of the 1.22-Mev decays, giving a partial lifetime lower limit of 2500 hours for positron emission to any of the excited levels in Sn^{118}. The level scheme is discussed in terms of various nuclear models.

EVIDENCE FOR COMPOUND NUCLEUS FORMATION USING (p,p') AND (α,p) SCATTERING IN NICKEL. Raymond Fox and Richard D. Albert, Lawrence Radiation Laboratory, University of California, Livermore, California (Received August 22, 1960).

Spectra for (p,p') and (α,p) reactions have been obtained by bombarding nickel targets with the beam of the Livermore 90-inch cyclotron and magnetically analyzing protons emitted at 135°. These reactions were studied for several energies of incident protons between 7.8 and 11.4 Mev, and incident alpha particles between 9.65 and 12.8 Mev. When the differential scattering cross section is divided by the emitted

proton energy and the black nucleus cross section for protons, results are obtained for different incident proton energies which have the same relative shape when plotted vs excitation energy. This is strong evidence for formation of compound nucleus in these reactions. A large peak is observed in the spectrum at 4.75 Mev, an excitation energy where the level density is sufficiently high that it is difficult to attribute this peak to a single level. An anomalous peak at about this energy has been previously observed for 23-Mev (p, p') scattering and 30-Mev (α, α') scattering on nickel targets by Cohen, and Sweetman and Wall. Results obtained for the (α, p) spectrum are in good agreement with predictions of the statistical model of the compound nucleus assuming a level density of the form $\exp[2(aE)^{1/2}]$.

GAMMA-RAY CORRELATIONS FROM THE REACTION $B^{10}(d, p)B^{11}$. Walter Reichelt* and George E. Owen, The Johns Hopkins University, Baltimore, Maryland (Received September 6, 1960; revised manuscript received October 13, 1960).

The $B^{10}(d, p)B^{11}$ reaction was studied at a deuteron bombarding energy of 1.2 Mev. Angular distributions and proton-gamma correlations associated with transitions to the first and second excited states of B^{11} were obtained. An analysis including the effects of heavy-particle stripping has been performed for the first excited state transition and is consistent with the observed distribution and correlation.

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MEASUREMENT OF THE ANNIHILATION-IN-FLIGHT CROSS SECTION AT 0° FOR 8.5-Mev POSITRONS. F. D. Seward, C. R. Hatcher, and S. C. Fultz, Lawrence Radiation Laboratory, University of California, Livermore, California (Received September 12, 1960).

The differential cross section at 0° for the annihilation-in-flight of 8.5-Mev positrons has been measured. The positrons were created in a thick Ta target which was bombarded by 20-Mev electrons from a linear accelerator. They were directed onto a Be target where annihilation occurred, and the annihilation photons were measured by use of a thick-crystal spectrometer. The measured value for the cross section is 1.3 ± 0.2 barns/steradian per electron, which is in agreement with theory.

RANGES AND RANGE STRAGGLING OF Tb^{149}, At, AND Po. Lester Winsberg and John M. Alexander, Lawrence Radiation Laboratory, University of California, Berkeley, California (Received September 15, 1960).

We report on ranges and range straggling of recoils from nuclear reactions induced by the ions C^{12}, N^{14}, O^{16}, O^{18}, and Ne^{22} with kinetic energies of 10 Mev per nucleon and less. Range-energy curves were obtained for Tb^{149} (recoil energies of 4 to 29 Mev) in Al, for At and Po (4 to 15 Mev) in Al, and for At and Po (4 to 9 Mev) in Au. Ranges of Tb^{149} at the threshold of each reaction were obtained by extrapolation. The agreement of these and the direcly measured values supports the assumption of compound-nucleus formation used in calculating the recoil energies. The smaller recoil velocities in this study are of the same order as the Bohr velocity $(2.2 \times 10^8$ cm/sec). The values of the average range and the straggling parameter are compared with stopping theory. The contribution to the measured range straggling from the nuclear reaction is discussed. Our results and the work of others are used to obtain values of the range for Xe^{139} in Al from 0.1 to 70 Mev and for At^{203} in Au from 0.01 to 10 Mev.

RECOIL STUDIES OF NUCLEAR REACTIONS INDUCED BY HEAVY IONS. John M. Alexander and Lester Winsberg, Lawrence Radiation Laboratory, University of California, Berkeley, California (Received September 15, 1960).

The mechanism of nuclear reactions induced by heavy ions (HI) was investigated by measuring the recoil ranges of Tb^{149}, At^{211}, and other alpha-emitting isotopes of At and neighboring elements and by determining the cross sections for the formation of Tb^{149} and At^{211}.

Recoil ranges were consistent with compound-nucleus formation at all energies studied for the following reactions: $Pr^{141}(C^{12}, 4n)Tb^{149}$, $Ce(N^{14}, xn)Tb^{149}$, $La^{139}(O^{16}, 6n)Tb^{149}$, $La^{139}(O^{18}, 8n)Tb^{149}$, and $Ba(Ne^{22}, pxn)Tb^{149}$. A similar result was obtained for the reaction $Pr^{141}(O^{16}, 2p6n)Tb^{149}$ at 138

and at 146 Mev and for the reactions $Au^{197}(O^{16}, 2pxn$ and $3pxn)At$, Po at energies below 100 Mev. The excitation functions of the $(HI, xn)Tb^{149}$ reactions seem to be characteristic of an evaporation process but have smaller peak cross sections than do the excitation functions of the reactions $Ba(Ne^{22}, pxn)Tb^{149}$ or $Pr^{141}(O^{16}, 2p6n)Tb^{149}$. We conclude that most reactions probably involve charged-particle emission. The reaction $Ba(Ne^{22}, pxn)Tb^{149}$ seems to occur with much greater probability than the reaction $Ba(Ne^{20}, pxn)Tb^{149}$.

In many cases the compound-nucleus mechanism cannot account for our results. Partial momentum transfer is observed in the reactions $Au^{197}(O^{16}, 2pxn$ and $3pxn)At$, Po at energies above 100 Mev. Partial momentum transfer also occurs when Bi is bombarded at energies 1.3 times the barrier energy or greater. Reactions of Bi with heavy ions (Ne^{20} is possible exception) at energies near the Coulomb barrier produce At^{211} with greater recoil energy than expected from a compound-nucleus mechanism. Apparently, particles are emitted in the backward direction. Near the barrier the cross section for the production of At^{211} by C^{12}, O^{16}, and Ne^{20} bombardment comprises about 1/4 the value calculated for compound-nucleus formation. Therefore, the cross section for all non-compound-nucleus reactions must comprise a large fraction of the total interaction cross section. The experiments with Pb as a target are also consistent with this conclusion.

NEUTRON-PROTON PAIRING INTERACTION. A. N. Saxena, Physics Department and High-Energy Physics Laboratory, Stanford University, Stanford, California (Received August 26, 1960; revised manuscript received October 24, 1960).

The neutron-proton pairing interaction λ between the last odd neutron and the last odd proton in the outermost neutron and proton shells of an odd-odd nucleus has been estimated from nuclear masses in the regions just beyond $Z = 20$, $N = 20$, and just beyond $Z = 40$, $N = 50$. Behavior of λ in these two regions and in the heavy-element region $Z > 82$, $N > 126$, as estimated by Ghoshal and Saxena, is discussed. It is found that the behavior of λ may be understood in terms of a simple jj-coupling shell model. According to this model, λ arises from the spin-independent part of the two-body force and is proportional to $(2j_1 + 1 - 2z) \times (2j_2 + 1 - 2n)$, where z is the odd number of protons in the outermost proton shell j_1, and n is the odd number of neutrons in the outermost neutron shell j_2 of the odd-odd nucleus.

CAPTURE-GAMMA DETERMINATION OF V^{52} LEVELS. J. E. Schwäger, Lawrence Radiation Laboratory, University of California, Livermore, California (Received July 13, 1960; revised manuscript received October 7, 1960).

Thermal-neutron capture-gamma radiations for the $V^{51}(n, \gamma)V^{52*}$ reaction have been observed with a fast-coincidence scintillation spectrometer recently developed for the Livermore 1-Mw pool-type reactor. The added-neutron binding energy in V^{52} was measured as 7.30 ± 0.05 Mev and a decay scheme was established which verifies a previously proposed $V^{51}(d, p)V^{52*}$ level scheme. Several low-energy crossover, without stopover, transitions were observed: two from the 0.78-Mev level to the ground and 0.13-Mev levels (the latter being the more intense) but not to the 0.42-Mev level, and one from the 0.83-Mev level to the ground state only. The 0.42-Mev level makes both crossover and stopover transitions to the ground and 0.13-Mev levels (the former being the stronger). Some evidence exists to suggest that the low-lying V^{52*} states arise from excitations of the $(1f_{7/2})^3$ proton configuration alone.

CAPTURE-GAMMA DETERMINATION OF V^{51} LEVELS. J. E. Schwäger, Lawrence Radiation Laboratory, University of California, Livermore, California (Received July 11, 1960).

A fast-coincidence scintillation spectrometer has been developed for thermal-neutron capture-gamma studies at the Livermore 1-Mw pool-type reactor. The added-neutron binding energy in V^{51} was measured as 11.1 ± 0.1 Mev. Cascade radiations for V^{51*} were observed and a decay scheme is established which verifies a previously proposed (p, p') level scheme. Spin and parity assignments for most of the $V^{51}(p, p')$ reported levels below 4.0 Mev are proposed and the level order for the $(1f_{7/2})^3$ proton configuration levels is established as $7/2^-$ (ground state), $5/2^-$, $3/2^-$, $11/2^-$, $9/2^-$, and $15/2^-$ with excitation energies of 0.32, 0.93, 1.61, 1.81, and 2.70 Mev, respectively.

Of the various nuclear force assumptions that can be made, the short-range force approximation (δ-type interaction) and weak surface-coupling effects for V^{51*} configuration states appear to give the best match between theory and experiment. A previous calculation using experimentally measured splittings of the j^2 configuration together with tabulated coefficients of fractional parentage gives excellent agreement with experiment. Evidence is also found for $(1f_{7/2}^2)_0\ 2p_{3/2}$- and $(1f_{7/2}^2)_0\ 1f_{5/2}$- proton single-particle levels at 2.41 and 2.55 Mev, respectively, as well as for additional low-spin states between 2.70 and 3.38 Mev which possibly represent even states resulting from excitation of a lower shell proton to give a $(1f_{7/2}^4)_0$ configuration plus a proton hole in the vacated shell. Experimental results suggest that the V^{51*} 20-28 core is a more rigid structure than the Ca^{43*} 20-20 core.

NUCLEAR MOMENT OF Ce^{137m} BY NUCLEAR ALIGNMENT. J. N. Haag, C. E. Johnson,* D. A. Shirley, and D. H. Templeton, Lawrence Radiation Laboratory and Department of Chemistry, University of California, Berkeley, California (Received September 2, 1960).

Nuclei of Ce^{137} and Ce^{137m} have been aligned at low temperatures in a single crystal of neodymium ethylsulfate nonahydrate by means of the magnetic hfs coupling with the electrons of the Ce^{+3} ions. The anisotropy of their gamma radiation has been observed. The magnetic moment of Ce^{137m} is $|\mu_N| = 0.96 \pm 0.09$ nm. The spin of Ce^{137m} is established as 11/2.

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RADIATIVE CAPTURE OF PROTONS IN C^{13}. Ross E. Hester and W. A. S. Lamb, Lawrence Radiation Laboratory, University of California, Livermore, California (Received August 22, 1960).

An excitation curve of the $C^{13}(p,\gamma)N^{14}$ reaction has been measured from 100 kev to 140 kev. The cross section ranges from $(7.7 \pm 1.8) \times 10^{-34}$ cm^2 at 100 kev to $(9.8 \pm 1.2) \times 10^{-33}$ cm^2 at 140 kev. The results are compared with those of previous measurements.

ELASTIC SCATTERING OF ALPHA PARTICLES FROM HELIUM. J. R. Dunning, A. M. Smith, and F. E. Steigert, Yale University, New Haven, Connecticut (Received September 9, 1960).

The elastic scattering of alpha particles from helium has been investigated at laboratory energies of 6.43, 6.84, and 7.78 Mev. Complete angular distributions from 20° to 90° in the center-of-mass system were obtained. Analysis of the data suggests somewhat smaller values for the D-wave phase shift than previously reported.

INELASTIC ELECTRON SCATTERING FROM THE ALPHA PARTICLE. G. R. Burleson,* Department of Physics and High-Energy Physics Laboratory, Stanford University, Stanford, California (Received September 6, 1960).

The inelastic scattering of electrons from He^4 which corresponds to a disintegration of the nucleus has been studied for incident electron energies of 400 and 500 Mev at laboratory angles from 45° to 135°. The energy spectra of the scattered electrons were measured, and absolute cross sections were found by comparison with elastic scattering from the proton. The curves were corrected for electron radiation. Within the validity of adapting to He^4 one of the results of the Goldberg theory of deuteron electrodisintegration, the cross sections at the maxima of the curves give a value of $M\langle 1/p\rangle_\alpha$ of (7.5 ± 1.5), where M is a nucleon mass and $\langle 1/p\rangle_\alpha$ is the expectation value of the reciprocal of the momentum of a nucleon bound in He^4. With a single exception, the energy-integrated cross sections $d\sigma_\alpha/d\Omega$ agree within experimental error with $d\sigma_\alpha/d\Omega = 2(d\sigma_p/d\Omega + d\sigma_n/d\Omega)$, where $d\sigma_p/d\Omega$ is the free proton cross section and $d\sigma_n/d\Omega$ is the neutron cross section found from inelastic scattering from the deuteron.

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SEARCH FOR NEUTRINOLESS CONVERSION OF MUON INTO ELECTRON. R. D. Sard,* K. M. Crowe, and Hans Kruger, Lawrence Radiation Laboratory, University of California, Berkeley, California (Received September 6, 1960).

A search has been made for the hypothetical reaction in which a muon near a nucleus is con-

verted into an electron without production of neutrinos. Negative muons were stopped in a copper target. A magnetic spectrometer at right angles to the beam transmitted particles of the momentum expected for the electron (about 90 Mev/c on entry into the spectrometer). A long scintillation counter at the output of the spectrometer gave a pulse corresponding to the emerging particle's energy loss. Selection by both momentum and pulse-height eliminated particles heavier than the electron and greatly reduced the accidental background. In the main run, three events meeting the selection criteria were recorded, while the expected number of accidentals is 0.23 ± 0.04. Various alternative processes that would produce accepted events are considered and found to give expectation values even smaller than that for accidentals. Without further experimentation, we cannot decide whether the hypothetical reaction does or does not occur, but we can set an upper limit of $(4^{+9}_{-2}) \times 10^{-6}$ on the ratio, R, of the reaction rate to that of normal absorption.

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EXTENSION OF THE VARIATIONAL METHOD FOR HARD-SPHERE BOSONS. Richard J. Drachman, Brandeis University, Waltham, Massachusetts, and Nucleonics Division, U.S. Naval Research Laboratory, Washington, D. C. (Received August 31, 1960).

The coordinate space variational treatment developed by Aviles and Iwamoto for the hard-sphere boson gas has been extended to include the first logarithmic term in the ground-state energy expansion. The result agrees, within four percent, with the exact results of Wu and others. The form of the variational wave function used is discussed and compared with that obtained using the pseudopotential method of Lee, Huang, and Yang.

SYMMETRY THEOREMS FOR ISOSPIN-INVARIANT REACTIONS. Murray Peshkin,* The Weizmann Institute of Science, Rehovoth, Israel (Received August 26, 1960).

Symmetry theorems, analogous to those well known for angular distributions and correlations, are given for isospin-invariant reactions starting from an initial state of limited complexity. Detailed calculations are carried out when the initial state isospin does not exceed 3/2. A statistical generalization is given for averages over experiments starting from different charge states. Some properties of the irreducible tensor operators which arise from recoupling the angular momentum operator to itself are discussed.

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